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A WELL-POSED PROBLEM FOR THE HEAT EQUATION

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ABSTRACT. Simultaneous specification of (consistent) Dirichlet and Neumann data boundedly determines later internal states of the solution of the heat equation in a general region.

We consider solutions of the heat equation $u_t=\Delta u$ for $0<t<T$, $x=(x_1, \cdots, x_n) \in \Omega$. It is well known that arbitrary specification of both the *initial state* $u_0=u(0, \cdot)$ and either *Dirichlet data*:

$$u(t, x) = f(t, x) \quad \text{for } 0 \leqq t \leqq T,\ x \in \partial\Omega,$$

or *Neumann data*:

$$(\partial u/\partial \nu)(t, x) = g(t, x) \quad \text{for } 0 \leqq t \leqq T,\ x \in \partial\Omega,$$

determines uniquely the evolution of the process. In particular, the *terminal state* $u_T=u(T, \cdot)$ is determined by either of the pairs (u_0, f), (u_0, g).

If the initial internal state is not given, we ask whether knowledge of *both* Dirichlet *and* Neumann data suffices. The pair (f, g) cannot be specified arbitrarily, but we adopt the viewpoint that in observation of an ongoing process, the consistency conditions are automatically satisfied so the observed pair (f, g) lies in the admissible manifold M, and the existence of a solution is not at issue. We ask whether *observation* of the boundary data (f, g) suffices for effective *prediction* of the terminal internal state u_T.

THEOREM. *The observation/prediction problem for the heat equation is well posed for any bounded region Ω in $\boldsymbol{R}^n$ with smooth boundary $\partial\Omega$. I.e., in the above notation, the map: $(f, g) \mapsto u_T$ is well defined and continuous, using appropriate $\mathscr{L}_2$ topologies for domain and range.*

SKETCH OF PROOF. (a) There is a reduction to a *restricted* problem of the same form with $g \equiv 0$.

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