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FACTORIZATION AND INVARIANT SUBSPACES FOR NONCONTRACTIONS

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1. **Introduction.** The purpose of this note is to announce a generalization of the Sz.-Nagy-Foiaş model theory for contractions to arbitrary bounded operators. We also indicate how invariant subspaces are described by this model theory.

The Russians, for example, Livšic [**14**] and Brodskiĭ and Livšic [**7**], have studied model theories for various classes of operators, often including some noncontractions. Recently there has been some work, for example, Davis and Foiaş [**13**], Brodskiĭ, Gohberg, and Kreĭn [**9**], and Brodskiĭ [**8**] on characteristic functions for noncontractions. Our work is closely related to that of Clark [**11**] and depends heavily for inspiration upon the canonical models of de Branges-Rovnyak [**5**].

In many of these papers, one of the main points is the connection between factorizations of the characteristic function B and invariant subspaces. Sz.-Nagy-Foiaş [**15**] found a precise condition on a factorization of B to insure that it results (for contractions) in an invariant subspace. Also the work of de Branges [**4**], [**5**] should be mentioned. Most recently Clark [**12**] has taken this problem up for invertible noncontractions. We propose to study this problem for the class of bounded noncontractions.

2. **Model theory.** The characteristic operator function $B(z)$ of a bounded Hilbert space operator T is defined by

$$B(z) = -TJ_T + z\,|I - TT^*|^{1/2}(I - zT^*)^{-1}\,|I - T^*T|^{1/2} \tag{1}$$

where $J_T = \operatorname{sgn}(I - T^*T)$ and where B acts from $\mathscr{D}_T$, the closure of the range of $|I - T^*T|^{1/2}$, to $\mathscr{D}_{T^*}$. A basic problem of model theory is to construct from B, in a canonical way, a bounded operator T such that B satisfies (1).

Let $\mathscr{C}_*$ and $\mathscr{C}$ be Hilbert spaces, and let $B(z)\colon \mathscr{C} \to \mathscr{C}_*$ be analytic in a neighborhood D of 0. We also assume that D is symmetric about the real line. Let

$$J = \operatorname{sgn}(I - B(0)^*B(0)), \qquad J_* = \operatorname{sgn}(I - B(0)B(0)^*), \qquad \operatorname{sgn} 0 = 1.$$

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