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MEASURABLE CHOICE AND THE INVARIANT SUBSPACE PROBLEM

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In [**1**], J. Dyer, A. Pedersen and P. Porcelli announced that an affirmative answer to the invariant subspace problem would imply that every reductive operator is normal. Their argument, outlined in [**1**], provides a striking application of direct integral theory. Moreover, this method leads to a general decomposition theory for reductive algebras which in turn illuminates the close relationship between the transitive and reductive algebra problems.

The main purpose of the present note is to provide a short proof of the technical portion of [**1**]: that invariant subspaces for the direct integrands of a decomposable operator can be assembled "in a measurable fashion". The general decomposition theory alluded to above will be developed elsewhere in a joint work with C. K. Fong, though we do present a summary of some of its consequences below.

All Hilbert spaces discussed in this paper will be separable and all operators will be bounded. We use the term 'algebra' to refer to an identity—containing algebra of operators which is closed in the weak operator topology. A *transitive* algebra is an algebra having no nontrivial invariant subspaces; more generally, an algebra is called *reductive* if it is reduced by each of its invariant subspaces.

The reader is referred to [**2**] or [**3**] for the details of direct integral theory; the primary purpose of the following summary is to fix notation. Let μ be the completion of a finite positive regular Borel measure supported on a σ-compact subset of a separable metric space Λ and let $\{e_n\}$, $1 \leqq n \leqq \infty$, be a collection of disjoint Borel subsets of Λ with union Λ. Let $h_1 \subseteq h_2 \subseteq \cdots \subseteq h_\infty$ be a sequence of Hilbert spaces with h_n having dimension n and h_∞ spanned by the remaining h_n's. We write $h = \int_\Lambda \oplus\, h(\lambda)\mu\,(d\lambda)$ for the Hilbert space of (equivalence classes of) weakly measurable functions f from Λ into h_∞ such that for $\lambda \in e_n$, $f(\lambda) \in h(\lambda) \equiv h_n$, and $\int_\Lambda \|f(\lambda)\|^2 \mu\,(d\lambda) < \infty$. The element in h represented by the function $\lambda \rightarrow f(\lambda)$ is denoted by $\int_\Lambda \oplus\, f(\lambda)\mu\,(d\lambda)$.

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