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# IRREDUCIBLE REPRESENTATIONS OF LIE ALGEBRA EXTENSIONS

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This note announces three density theorems involving representations of Lie algebras and associative algebras. The first theorem describes the irreducible (possibly infinite dimensional) representations $\rho$ of a Lie algebra $\mathfrak{g}$ with an ideal $\mathfrak{k}$ such that the restriction of $\rho$ to $\mathfrak{k}$ has some absolutely irreducible quotient representation. The second result is an embedding theorem for the irreducible representations of the Weyl algebras $A_{n,C}$ over $C$ ($A_{n,C} \cong C[t_1, \cdots, t_n, \partial/\partial t_1, \cdots, \partial/\partial t_n]$, the associative algebra of partial differential operators on $n$ variables with coefficients in the polynomial ring $C[t_1, \cdots, t_n]$). Our result is a sort of algebraic analogue of the uniqueness of the Heisenberg commutation relations, and has an application to irreducible representations of nilpotent Lie algebras via Dixmier's theory [**5**]. The third theorem describes the differentiably simple algebras having a maximal ideal. This result unifies the author's theorem [**3**] on differentiably simple rings with a minimal ideal, and Guillemin's theorem [**7**], [**2**] on the structure of a nonabelian minimal closed ideal of a linearly compact Lie algebra.

1. In what follows, all algebras, tensor products etc., will be over an arbitrary given field $\Phi$, unless otherwise stated. If the characteristic is prime, the Lie algebras considered will always be assumed restricted (=Lie $p$-algebra), and the same for their homomorphisms, ideals, etc. Also $U$ will denote the universal enveloping algebra functor at characteristic 0, and the restricted universal enveloping algebra functor at prime characteristic. We shall take $\mathfrak{g}$ to be a given Lie algebra, and $\mathfrak{k}$ an ideal of $\mathfrak{g}$.

Recall that if $V$ is a $\mathfrak{k}$-module with corresponding representation $\sigma$, then the stabilizer $\mathrm{St}(V, \mathfrak{g})$ of $V$ in $\mathfrak{g}$ is defined [**1**], [**6**] by

$$\mathrm{St}(V, \mathfrak{g}) = \{x \in \mathfrak{g} \mid \exists \eta \in \mathrm{Hom}(V, V) \ni \sigma[x, y] = [\eta, \sigma y] \forall y \in \mathfrak{k}\}.$$

This is a subalgebra of $\mathfrak{g}$ containing $\mathfrak{k}$, and gives the analogue of the concept of stabilizer for group representations.

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