BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 80, Number 5, September 1974

HIGHER DIFFERENTIAL ALGEBRAS OF DISCRETE VALUATION RINGS

BY SATOSHI SUZUKI AND JUN-ICHI NISHIMURA

Communicated by Dock Rim, December 12, 1973

Let S be a commutative ring with identity. Let T be a commutative S-algebra. $A_S(T)$ denotes the higher differential algebra over S, in the sense of Berger [1] or Kawahara-Yokoyama [5], with an index set consisting of all nonnegative integers. $\{d^n_{T/S}\}_{n=0,1,2,\cdots}$ denotes the canonical higher S-derivation of T into $A_S(T)$. In case S is the ring of all integers, we use simplified notations $A(T)$ and d^n_T $(n=0, 1, 2, \cdots)$.

Let R denote a complete discrete valuation ring, of a valuation v of unequal characteristic with maximal ideal $\mathfrak{m}=(\pi)$. Assume that the characteristic of $k=R/\mathfrak{m}$ is p. Let P be a coefficient ring of R. Let $\{\bar{c}_\iota\}_{\iota\in\Gamma}$ be a p-independent base of k and let c_ι be a representative of $\bar{c}_\iota$ chosen from P for every $\iota\in\Gamma$. The symbol $\hat{}$ means the p-adic completion of P-algebra. By arguments developed by Berger or Kawahara-Yokoyama in the cited papers, and formal smoothness and flatness of P over the prime local ring, we can deduce the following theorem.

THEOREM 1. *$A(P)^\wedge = P[d^n_P c_\iota]^\wedge_{\iota\in\Gamma;n=0,1,2,\cdots}$, where $P[d^n_P c_\iota]_{\iota\in\Gamma;n=0,1,2,\cdots}$ is a polynomial ring over P in distinct indeterminates $d^n_P c_\iota$'s.*

For simplicity, we denote canonical images of $d^n_R c_\iota$ in $A(R)^\wedge$ by the same notation $d^n_R c_\iota$, for $\iota\in\Gamma$. Let $\{d^n\}_{n=0,1,2,\cdots}$ be the canonical higher derivation of the polynomial ring $P[X]$ into $(R\otimes_{P[X]} A(P[X]))^\wedge$. Let $f(X)$ be the Eisenstein polynomial over P such that $f(\pi)=0$. Then we have the following formula for every $n\geqq 1$.

$$d^n f(X) = f'(\pi)\, d^n X + \sum_{j\geqq 2} \frac{f^{(j)}(\pi)}{j!} \sum d^{i_1}X \cdots d^{i_j}X + pG_n(d^1X, d^2X, \cdots; \cdots, d^i c_\iota, \cdots), \tag{1}$$

where the second sum of the middle term is taken for the sets of integers

---

AMS (MOS) subject classifications (1970). Primary 13B10, 13H05, 13J10; Secondary 13A15, 13B15.

Key words and phrases. Higher differential algebra, complete discrete valuation ring, Eisenstein polynomial, coefficient ring.