

A REPRESENTATION OF CLOSED, ORIENTABLE
3-MANIFOLDS AS 3-FOLD BRANCHED
COVERINGS OF S^3

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A *represented link* is a link L in S^3 together with a representation ω of the link group $\pi(S^3 - L)$ into the symmetric permutation group of d symbols \mathcal{S}_d . Let us call ω *simple* if it represents each meridian of L by an appropriate transposition. If (L, ω) is a represented link, there is a uniquely associated closed, orientable 3-manifold $M(L, \omega)$, namely the d -fold covering of S^3 branched over L , that is determined by the representation ω . It has been proved by J. W. Alexander [1] that every closed orientable 3-manifold is $M(L, \omega)$ for some link L and representation ω .

H. M. Hilden (personal communication to the author) has proved

THEOREM 1. *Every closed, orientable 3-manifold is $M(K, \omega)$ for some knot K and simple representation ω of $\pi(S^3 - K)$ onto \mathcal{S}_3 .*

Theorem 1 states that every closed, orientable 3-manifold is a 3-fold irregular covering space of S^3 branched over a knot K in such a way that the inverse image of a point of K consists of a point of branch-index 2 and a point of branch-index 1.

We have obtained (independently) a different proof of Theorem 1 which will be sketched here. A detailed proof will appear elsewhere.

Let L be a link in S^3 composed of two unlinked trivial knots P and R , and let ω be a representation of $\pi(S^3 - L)$ onto the group \mathcal{S}_3 of permutations of the symbols 0, 1, and 2, such that ω represents a meridian of P (resp. R) by the transposition (01) (resp. (02)). It is easy to see that $M(L, \omega)$ is S^3 . Let $p: M(L, \omega) \rightarrow S^3$ be the covering projection. Then $p^{-1}(P)$ (resp. $p^{-1}(R)$) is composed of a curve P_{01} (resp. R_{02}) of branch-index 2 and a curve P_2 (resp. R_1) of branch-index 1. These curves are unknotted and unlinked. Let B be a ball in S^3 which cuts L in exactly two disjoint arcs in P and such that $p^{-1}(B)$ is the disjoint sum of a ball B_2 which cuts P_2 , and a solid torus B_{01} which cuts P_{01} . Then $p|_{B_{01}: B_{01} \rightarrow B}$ is

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