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A UNIFIED APPROACH TO GENERALIZED INVERSES OF LINEAR OPERATORS: I. ALGEBRAIC, TOPOLOGICAL AND PROJECTIONAL PROPERTIES

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0. **Introduction.** In this announcement we abstract a theory for generalized inverses (G.I.) of a linear operator between two algebraic linear spaces, or topological vector spaces. Our approach includes as special cases all previously published approaches to G.I.'s of matrices and linear operators in Hilbert spaces. In addition it provides new results for G.I.'s in the case of normed spaces, and develops the first treatment of G.I.'s of arbitrary linear operators between two topological vector spaces. The explicit transformation of G.I.'s under changes of projectors is new even in the case of matrices. For a survey of various definitions and results on G.I.'s of linear operators, see Nashed [7] and the references cited therein; see also the papers by Erdelyi [3], Erdelyi and Ben-Israel [4], Deutsch [2], and Koliha [6], which appeared after [7]. Other recent references are cited in [8].

The proofs of these results and a detailed account of the various special cases alluded to will appear elsewhere [8].

1. **Algebraic and projectional properties.** Let V and W be vector spaces over the same (real or complex) field, and let L be a linear map from V into W. The range and null spaces of L are denoted by $\mathscr{R}(L)$ and $\mathscr{N}(L)$ respectively. A linear map M from W into V such that $LML=L$ is called a *partial* (or *inner*) inverse of L. Every linear map has a partial inverse. If M is a partial inverse of L, then LM and ML are idempotent, $\mathscr{R}(ML)\subset \mathscr{R}(M)$, $\mathscr{R}(LM)=\mathscr{R}(L)$, $\mathscr{N}(M)\subset\mathscr{N}(LM)$, and $\mathscr{N}(ML)\subset\mathscr{N}(L)$. A partial inverse M determines a particular complementary subspace to

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