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LEBESGUE SPACES FOR BILINEAR VECTOR INTEGRATION THEORY

BY JAMES K. BROOKS[1] AND NICOLAE DINCULEANU[2]

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In this note we shall announce results concerning the structure of $L_E^1(m)$, the space of E-valued functions integrable with respect to a measure $m:\Sigma\to L(E, F)$, where $L(E, F)$ is the class of bounded operators from the Banach space E into the Banach space F. The bilinear integration theory introduced here is more restrictive than the one developed by Bartle **[1]**, but it is general enough to allow a norm to be defined on the integrable functions and to permit the study of weak compactness and convergence theorems; moreover, $L_E^1(m)$ lends itself in a natural way to the study of continuous operators $T: C_E(S)\to F$, where the domain is the space of continuous E-valued functions defined on the compact Hausdorff space S as follows: By Dinculeanu's representation theorem **[6]**, there exists a unique regular finitely-additive measure $m:\Sigma\to L(E, F^{**})$, where Σ is the family of Borel subsets of S, such that $T(f)=\int f\,dm$. If T is a weakly compact operator, Brooks and Lewis **[2]** have shown that m is countably additive, with range in $L(E, F)$. In addition, the set $N=\{|m_z|:z\in F_1^*\}$ is relatively weakly compact in ca(Σ)—here m_z is the E^*-valued measure defined by $m_z(A)e=\langle m(A)e, z\rangle$, and $|m_z|$ is the total variation function of m_z. Conversely, if N has the above property and E is reflexive, then T is weakly compact. A natural question is whether a Lebesgue space $L_E^1(m)\supset C_E(S)$ of m-integrable functions can be defined. If so, what convergence theorems can be proved, and how are the weakly compact sets characterized?

The setting is as follows. Let Σ be a σ-algebra of subsets of a set T, and $m:\Sigma\to L(E, F)$, a countably additive measure be given such that m is strongly bounded, that is, $\tilde{m}_{E,F}(A_i)\to 0$, whenever (A_i) is a disjoint sequence of sets ($\tilde{m}_{E,F}$ is the semivariation of m with respect to E and F **[6]**). It follows that $N=\{|m_z|:z\in F_1^*\}$ is relatively weakly compact in ca(Σ). Let λ be a positive control measure for m such that $\lambda\leqq\tilde{m}_{E,F}$ and

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