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MODULI OF MARKED RIEMANN SURFACES

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The purpose of this note is to exhibit a set of complex analytic moduli for the space of closed Riemann surfaces of genus $g \geqq 2$, marked by a basis for the fundamental group. That one could find such moduli (i.e., biholomorphically embed the Teichmüller space of genus g in C^{3g-3}) was proven by Bers [2]. His moduli are variational—they depend on a choice of base surface. Our moduli are in some sense intrinsic, similar to the (real) moduli of Fenchel-Nielsen [4] and Keen [5]. In fact our moduli should be regarded as the complex analogue of the Fenchel-Nielsen and Keen moduli. (The geometric relationship between these different moduli is clear, and they are real-analytically equivalent.)

The actual expressions for the moduli given below involve multiplicative constants and square roots. These normalizations serve two purposes. First, the moduli space is contained in a product of half-planes. Second, with these normalizations, the group of translations

$$(1) \qquad z \to z + n, \qquad n = (n_1, \cdots, n_{3g-3}) \in Z^{3g-3},$$

is a subgroup of the modular group; i.e., two points of the space of moduli which are identified under (1) correspond to the same Riemann surface with different markings.

The moduli occur as moduli of a set of generators of a Kleinian group; these are defined by traces of loxodromic elements and cross-ratios of fixed points of parabolic elements. Each of the $3g-3$ coordinates is determined by a subgroup of the Kleinian group; using this one sees that each of the coordinates can be identified, in a natural way, with the modulus of a torus.

In this note we present proofs only in very broad outline—details will appear elsewhere.

1. Let S be a closed Riemann surface of genus $g \geqq 2$, and let $A_1, B_1, \cdots, A_g, B_g$ be a canonical homotopy basis on S (we regard $A_1, \cdots, B_g$ as being both a set of loops on S and as a set of generators for $\pi_1(S)$).

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