BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 80, Number 4, July 1974

SINGULARITIES AND BORDISM OF q-PLANE FIELDS AND OF FOLIATIONS[1]

BY ULRICH KOSCHORKE

Communicated by Glen E. Bredon, January 3, 1974

1. **Introduction.** Let $\mathfrak{P}\mathfrak{N}_n(q)$ (resp. $\mathfrak{P}\mathfrak{N}_n^{or}(q)$) be the bordism group of n-dimensional smooth manifolds with arbitrary (resp. oriented) q-plane fields, and let $\mathfrak{P}\Omega_n(q)$ and $\mathfrak{P}\Omega_n^{or}(q)$ denote the corresponding groups based on oriented manifolds. In this paper we present a method which allows us in many cases to determine these groups. We use the forgetful homomorphism $f_{\mathfrak{P}}: \mathfrak{P}\mathfrak{N}_n(q) \to \mathfrak{N}_n(BO(q))$ (resp. $f_{\mathfrak{P}}: \mathfrak{P}\mathfrak{N}_n^{or}(q) \to \mathfrak{N}_n(BSO(q))$, resp. $f_{\mathfrak{P}}: \mathfrak{P}\Omega_n^{(or)}(q) \to \Omega_n(B(S)O(q))$), which assigns to the bordism class of a q-plane field the bordism class of (a classifying map of) the underlying vector bundle. Our point of departure is the following observation. If ξ is a q-dimensional vector bundle over an n-manifold M and $n \geqq 2q-3$, then it is always possible to find a vector bundle homomorphism $h: \xi \to TM$ which is injective outside of a $(q-1)$-dimensional submanifold S of M, and such that the kernel of h is 1-dimensional at every point of S. We investigate the behavior of h at such a singularity and obtain criteria as to when it is possible to cancel S without getting out of the original bordism class.

If M is closed and ξ is isomorphic to a q-dimensional subbundle of TM, then the element $TM-\xi$ in the K-theory of M can be represented by an $(n-q)$-dimensional bundle, and hence the class $[M, \xi]$ in the bordism of $B(S)O$ satisfies the following *vanishing condition*:

(V) all Whitney numbers of $[M, \xi]$ containing some $w_i(TM-\xi)$, $i > n-q$, as a factor, vanish.

Conversely we obtain

THEOREM 1. *Let $n > 2q-2$. Then under all four orientedness assumptions $[M, \xi]$ lies in the image of $f_{\mathfrak{P}}$ if and only if condition* (V) *is satisfied. Furthermore, the kernel as well as the cokernel of $f_{\mathfrak{P}}$ are finite groups consisting entirely of elements of order* 2.

A stable version of the first statement for the case of $\mathfrak{N}_n(BO(q))$ has previously been obtained by R. Stong [11] by other methods.

AMS (MOS) subject classifications (1970). Primary 57D25, 57D30, 57D90, 58A30; Secondary 55G35, 57D45.

[1] Research partially supported by NSF Grant GP-38215.