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BILINEAR FORMS AND CYCLIC GROUP ACTIONS

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In a recent paper [2] Conner and Raymond have given an approach to the study of smooth cyclic group actions which employs rational bilinear forms. If $K^{4n-1}=\partial B^{4n}$ bounds a compact oriented smooth manifold, there is a symmetric nonsingular rational bilinear form on the image of $H^{2n}(B, K; Q)\to H^{2n}(B; Q)$ which represents an element $w(B)$ in $W(Q)$, the rational Witt ring. Denoting the signature of this form by $\operatorname{sgn}(B)$ and the unit of $W(Q)$ by $\mathbf{1}$, the *peripheral invariant* of K,

$$\operatorname{per}(K) = w(B) - \operatorname{sgn}(B)\cdot\mathbf{1},$$

lies in the kernel of the signature homomorphism $\Phi: W(Q)\to Z$ and is independent of the choice of B. In [2] there is associated with any orientation preserving diffeomorphism (T, M^{4n}) of prime period p on a closed manifold an element of the kernel of Φ which we denote by $q(T, M)$, an invariant of the equivariant bordism class which vanishes on fixed point free actions. Using the peripheral invariant, Conner and Raymond computed $q(T, M)$, for $p=2$ or 3, in terms of the fixed point information. The fundamental problem posed in [2] is the extension of this result to all primes.

In this paper we give the general formula for all primes and apply it to establish relationships between the index of M and the index of the fixed set. The essence of the proof is a group isomorphism between the kernel of Φ and $\bigoplus_p W(Z_p)$ where $W(Z_p)$ is the Witt group of the field Z_p and the sum ranges over all primes. Using this isomorphism, we establish a relation between the peripheral invariant and the linking form which enables us to extend the definition of $\operatorname{per}(K)$ to any closed oriented $(4k-1)$-manifold.

1. **Bilinear forms.** Let $B\,\mathrm{Fin}$ denote the semigroup of isomorphism classes of symmetric nonsingular bilinear forms on finite abelian groups taking values in Q/Z. Denote by $W^s(Z)$ the semigroup of stable equivalence classes of nondegenerate integral bilinear forms on finitely

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