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A CLASS OF SQUARE-INTEGRABLE IRREDUCIBLE UNITARY REPRESENTATIONS OF SOME LINEAR GROUPS OVER COMMUTATIVE p-FIELDS[1]

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Let k be a commutative p-field which is not necessarily of characteristic 0 (cf. [2]). We denote by $\mathcal{O}$, $\mathcal{P}$ and $\mathcal{O}^\times$ the unique maximal compact subring of k, the unique maximal ideal contained in $\mathcal{O}$ and the group of invertible elements of the ring $\mathcal{O}$, respectively. Then the residue class field $\mathcal{O}/\mathcal{P}$ is a finite field of characteristic p ($p>1$ being a prime number). Let $n>1$ be a fixed positive integer. Let G be the subgroup of $\mathrm{GL}(n, k)$ consisting of those elements whose determinant belongs to $\mathcal{O}^\times$. Then $K=\mathrm{GL}(n, \mathcal{O})$ is a maximal compact subgroup of G.

Recently Shintani [1] constructed some square integrable irreducible unitary representations of G which are induced by suitable irreducible unitary representations of K, where these representations of K can be 'parametrized' by certain characters of suitable compact Cartan subgroups of G which are contained in K. However this construction is based on the assumption that n and p are relatively prime.

In the present note we show that an interesting subclass of these representations of G can be constructed without this assumption. Moreover we give a more explicit description of the structure of these representations of G in this case.

Formulation of the main result. Let k be a commutative p-field. Let $\mathcal{O}$, $\mathcal{P}$, $\mathcal{O}^\times$ be the maximal compact subring of k, the maximal ideal in $\mathcal{O}$ and the group of units in $\mathcal{O}$, respectively. Let π be a prime element of k and let q be the module of k. Then the residue class field $\tilde{k}=\mathcal{O}/\mathcal{P}$ is a finite field of characteristic p ($p>1$) containing q elements. For every $\nu \in Z$, we write $\mathcal{P}^\nu=\pi^\nu\mathcal{O}$ where $\mathcal{P}^0=\mathcal{O}$. Let $n>1$. Let G be the subgroup of the group $\mathrm{GL}(n, k)$ consisting of those elements whose determinant belongs to $\mathcal{O}^\times$. Let $K=\mathrm{GL}(n, \mathcal{O})$. Then G is a unimodular locally compact topological group and K is a maximal compact subgroup of G. We also note that the group G is totally disconnected. For every positive integer $m\geqq 1$, we set $K_m=\{x \in K: x\equiv I_n \pmod{\mathcal{P}^m}\}$. Then K_m is a compact open normal subgroup of finite index in K and moreover $K_1\supset K_2\supset\cdots$ form a fundamental system of neighborhoods of the identity I_n in K.

Let k' be an unramified extension of k of degree n over k. Let $\mathcal{O}'$,

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