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CHARACTERS OF CONNECTED LIE GROUPS[1]

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Introduction. If G is a finite group, any irreducible unitary representation of G gives rise to a homomorphism of the group algebra A (=formal linear combinations of the group elements with complex coefficients) the kernel of which is a 2-sided prime ideal, onto the full matrix algebra of the same dimension, and conversely. In this fashion, there is a canonical bijection between the set of all 2-sided prime ideals of A and characters of G. Let now G be a separable locally compact group. As generalization to this case of the group algebra we take the group C^* algebra $C^*(G)$ (cf. [**3**, 13.9, p. 270]), and as characters, following the definition, inspired by the pioneering work of R. Godement [**6**], [**7**], of A. Guichardet, the characters of $C^*(G)$ (cf. [**8**] or [**3**, 6.7, p. 126], and 1 below). Then every closed 2-sided prime ideal is primitive, or is the kernel of a factor representation, and conversely (cf. [**2**], Corollaires 1 and 3, p. 100]). Denoting by $\mathrm{Prim}(G)$ the set of all primitive ideals of $C^*(G)$, the question one is led to ask is whether the correspondence character $\mapsto$ primitive ideal establishes a bijection between the set of characters and $\mathrm{Prim}(G)$. By virtue of results of J. Glimm (cf. [**5**] or [**3**, 9.1, Théorème, (i)$\Rightarrow$(iv), p. 169]), in particular, the answer is yes for any type I group G. On the other hand Guichardet showed that countable infinite groups, in general, fail badly to have the said property [**8**, Proposition 2, p. 62].

The principal result of this note (cf. Theorem 1 below) states that the answer is again positive for any connected Lie group G. As a corollary, it solves the problem of the existence of characters, established until now only in an incomplete fashion for some special cases, as that of unimodular or solvable groups (cf. [**3**, 18.7.9, p. 326] and [**9**, Corollary 7.2, p. 594]). Our proof implies also that if G is solvable and simply connected, the factor representations constructed in [**9**, Theorem 2, p. 551]) by extending the geometrical approach, applied in the type I case by L. Auslander and B. Kostant [**1**] indeed provide the set of characters of G, as conjectured by the author (cf., e.g., [**9**, p. 463]). We believe that the essence of the

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