BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 80, Number 4, July 1974

AUTOMORPHIC MAPPINGS IN R^n

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Communicated by F. W. Gehring, December 12, 1973

1. By an automorphic mapping in R^n we mean a continuous, open, discrete, and sense-preserving mapping f from a domain D in R^n into $\bar{R}^n = R^n \cup \{\infty\}$ which satisfies $f \circ g = f$ for all $g \in G$ for some discrete group G of n-dimensional Möbius transformations, $n \geqq 2$. The results presented here indicate differences (see §5) as well as similarities (see §4) between automorphic functions in C and automorphic mappings of bounded dilatation in R^n, $n>2$. By mappings of bounded dilatation we mean *quasimeromorphic* (qm) mappings (cf. [**MRV** 1–2]).

2. Let G be a discrete Möbius group acting on the unit ball B^n. For $x_0 \in B^n$ which is not fixed by any element of $G\backslash\{\mathrm{id}\}$ the set $P = \{x \in B^n: d(x, x_0) < d(x, g(x_0)),\ \forall\, g \in G\backslash\{\mathrm{id}\}\}$ is a *normal fundamental polyhedron*; d denotes the hyperbolic distance. If the hyperbolic measure $V(B^n/G)$ of B^n/G is finite, then every normal fundamental polyhedron P has a finite number of $(n-1)$-faces and a finite number of *boundary vertices* $\{p_1, \cdots, p_k\} = \bar{P} \cap \partial B^n$ [S]. The last set is void when B^n/G is compact. P is said to be *simple* if for every boundary vertex $p \in \bar{P} \cap \partial B^n$ all the $(n-1)$-faces of P which meet at p are pairwise G-equivalent. By a recent result of Leon Greenberg (unpublished) it can be shown [MS] that if $V(B^n/G) < \infty$, then every point $b \in \partial B^n$ which is fixed by a parabolic element $g \in G$ is a boundary vertex of some simple fundamental polyhedron. A Möbius transformation is called *parabolic* if it has a unique fixed point in $\bar{R}^n$.

Complete proofs of the following theorems and related results will appear in [MS].

3. The existence of automorphic meromorphic functions for Möbius groups in C is usually proved by methods which cannot be used in R^n, $n>2$. However, with a suitable modification of a construction by J. W. Alexander [A] we obtain

THEOREM 1. *Every discrete Möbius group acting on B^n with $V(B^n/G) < \infty$ has qm automorphic mappings.*

AMS (MOS) subject classifications (1970). Primary 30A60; Secondary 30A68, 57A99, 31B15.