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A LOWER ESTIMATE FOR EXPONENTIAL SUMS

BY C. A. BERENSTEIN[1] AND M. A. DOSTAL

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1. **Introduction.** In this note we present two theorems on exponential sums (see Theorems 1 and 2 below). Although seemingly unrelated, both results are motivated by the study of a certain type of lower estimates of exponential sums in the complex domain. Thus while Theorem 2 is related to the validity of this estimate for all *discrete* exponential sums[2], Theorem 1 essentially says that even a milder estimate of this kind does not hold for a whole class of *continuous* exponential sums (i.e. for certain Fourier transforms).

In addition to the usual notation of the theory of distributions (cf. [2], [3], [7]), the following symbols will be used throughout this note. Given a distribution $\Phi \in \mathscr{E}' = \mathscr{E}'(\boldsymbol{R}^n)$, the symbol $[\Phi]$ ($\{\Phi\}$ resp.) denotes the convex hull of the support of Φ (singular support of Φ, resp.). For $A \subset \boldsymbol{R}^n$, h_A is the supporting function of A, i.e. $h_A(\lambda) = \sup_{x\in A} \langle x, \lambda\rangle$, $\lambda \in \boldsymbol{R}^n$. For $\zeta \in \boldsymbol{C}^n$ and $r > 0$, $\Delta = \Delta(\zeta; r)$ is the closed polydisk with center ζ and radius r; and, if $g(\zeta')$ is any continuous function on $\Delta(\zeta; r)$, we shall write

$$|g(\zeta)|_r = \max_{\zeta' \in \Delta} |g(\zeta')|. \tag{1}$$

2. **Indicators of smooth convex bodies.**

DEFINITION. Let $\Phi \in \mathscr{E}'$ be such that

$$\{\Phi * \Psi\} = \{\Phi\} + \{\Psi\} \qquad (\forall \Psi \in \mathscr{E}'). \tag{2}$$

Then Φ will be called a *good convolutor*.

The relationship of being a good convolutor to the solvability of the convolution equation $\Phi * u = f$ in the appropriate distribution spaces was discovered by L. Hörmander [7], and since then it was discussed by several authors (for references, cf. [2, Chapter I]). However, it is usually not easy to decide whether a given distribution Φ is a good convolutor or not.

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[2] And more generally, for all exponential polynomials.