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POSITIVE DEFINITE FUNCTIONS AND VOLTERRA INTEGRAL EQUATIONS[1]

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1. **Introduction.** The purpose of this research announcement is to describe a new approach for studying asymptotic behavior of solutions of functional equations involving a Volterra operator. More specifically, we study the role played by positive definite and related classes of functions as convolution kernels of the Volterra operators.

2. **Positive and D-positive definite functions.** Let $a(t) \in C(0, \infty) \cap L^1(0, 1)$. We say that $a(t)$ is *positive definite* if for any function $\varphi(t) \in C[0, \infty)$, the quadratic form

$$Q_a[\varphi](T) = \int_0^T \varphi(t) \int_0^t a(t-\tau)\varphi(\tau)\, d\tau\, dt \geqq 0, \qquad T \geqq 0. \tag{1}$$

Similarly, we say that $a(t)$ is *D-positive definite* if the quadratic form

$$R_a[\varphi](T) = \int_0^T \varphi(t) \frac{d}{dt} \int_0^t a(t-\tau)\varphi(\tau)\, d\tau\, dt \geqq 0, \qquad T \geqq 0. \tag{2}$$

This definition of positive definite functions differs slightly from that of Bochner since $a(0_+)$ is not assumed to exist and remains finite. The present form, as applied to the study of Volterra integral equations, was first introduced by Halanay [**1**], although he assumed that $a(t) \in C[0, \infty)$, thereby excluding the interesting case $t^{-\nu}$, $0<\nu<1$, the "so-called" Abel kernels. The idea of D-positive definite functions may be found in MacCamy [**6**] although his definition on $a(t)$ is even more restrictive. There is some ambiguity as to what $R_a[\varphi](T)$ means when $a(0_+)$ does not exist. This

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