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THREE STRUCTURE THEOREMS IN SEVERAL COMPLEX VARIABLES

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The purpose of this article is to describe three recent structure theorems in the theory of several complex variables and to point out a few of the many applications of these three theorems. In the first section we discuss a characterization of those currents (defined on an open subset of C^n) which correspond to integration over complex subvarieties. The second section is concerned with the structure of positive, d-closed currents. Finally, in the third section, a characterization of boundaries of complex subvarieties of C^n is discussed. A common thread in the techniques of proof involves "potential theory" for several complex variables.

1. **Recognizing currents that correspond to integration over complex subvarieties.** Suppose V is a complex subvariety of an open set in C^n with each irreducible component of V of dimension k. It is sometimes useful to consider, instead of the point set V, the linear functional "integration over V", which we denote by $[V]$. More precisely, for each compactly supported smooth form φ of degree $2k$, define $[V](\varphi)$ by integrating φ over the manifold points of V. A basic fact about complex subvarieties is that in a neighborhood of a singular point the $2k$-volume of the manifold points is finite (see **[4]**, **[16]**, or **[24]**). Therefore $[V](\varphi)$ is locally estimated by a constant times the supremum of the coefficients of the form φ. This implies that $[V]$ is a current (of real dimension $2k$ or (real) degree $2n-2k$). In fact, this estimate implies that the current $[V]$ viewed as a differential form with distribution coefficients actually has measures for coefficients.

There are several ways of recognizing which currents are of the form $[V]$ where V is a complex subvariety. The most elementary result of this kind says that if V is a real $2k$ dimensional submanifold of $C^n \cong R^{2n}$ and at each point of V the tangent space to V, considered as a real linear subspace of $R^{2n} \cong C^n$, is in fact a complex linear subspace, then V is a complex submanifold. It will shed light on later results to reinterpret this elementary result as follows. Suppose u is a current of degree $2(n-k)$ (dimension $2k$)

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