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THE METHOD OF EXTREMAL LENGTH

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Extremal length has become a useful tool in a wide variety of areas. The roots of the method can be traced back to the length-area comparisons in L. Ahlfors [1] and S. Warschawski [3], to the strip method of H. Grötzsch [1]–[15], and to even earlier works (see J. Jenkins [4, p. 7ff.] for a more complete historical background). In the 1940's Ahlfors and Beurling refined[2] those methods in a profound way; extremal length was introduced as a conformally invariant measure of curve families. This development appeared in Ahlfors-Beurling [4].

Subsequent applications of extremal length have been numerous and varied. There are geometric applications, function-theoretic applications, and applications which relate function-theoretic properties to geometric ones. In addition, there is a characteristically computational aspect which arises in connection with certain classical problems on univalent functions. I wish to discuss examples in each of these areas. There are many important topics I shall omit (prime ends, quasiconformal mapping, generalized modulus, and generalized capacity to mention a few); the extended bibliography will give a more complete picture.

1. **The concept of extremal length.** Let R be a surface (a surface, here, will be required to be connected, orientable, and to satisfy the second axiom of countability) with Riemannian metric ds_0. An important special case will be a region R in the plane endowed with the Euclidean metric.

A second metric ds on R is said to be *conformally equivalent* to ds_0 if these two metrics give rise to the same angular measure in each tangent space of R. This condition is also expressed by the requirement that at each point of R the metrics are proportional: $ds=\rho\, ds_0$ where ρ is a positive function on R.

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[2] For a glimpse of this development through the eyes of one of the principals, see the recent book L. Ahlfors [9, pp. 50, 81].