BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 80, Number 3, May 1974

STABLE MINIMAL SURFACES

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Communicated by S. S. Chern, May 1, 1973

Let $M \subset R^3$ be a minimal surface. A *domain* $D \subset M$ is an open connected set with compact closure $\bar{D}$ and such that its boundary ∂D is a finite union of piecewise smooth curves. We say that D is *stable* if D is a minimum for the area function of the induced metric, for all variations of $\bar{D}$ which keep ∂D fixed. In this note we announce the following estimate of the "size" of a stable minimal surface. We will denote by S^2 the unit sphere of R^3.

THEOREM. *Let $g: M \subset R^3 \to S^2$ be the Gauss map of a minimal surface M and let $D \subset M$ be a domain. If* area $g(D) < 2\pi$ *then D is stable.*

REMARK. The estimate is sharp, as can be shown, for instance, by considering pieces of the catenoid bounded by circles C_1 and C_2 parallel to and in opposite sides of the waist circle C_0. By choosing C_1 close to C_0 and C_2 far from C_0, we may obtain examples of unstable domains whose area of the spherical image is bigger than 2π and as close to 2π as we wish.

REMARK. The theorem implies that if the total curvature of D is smaller than 2π, then D is stable. The theorem is however stronger since we only use the area of the spherical image and the total curvature is equal to this area counting multiplicity.

REMARK. Our theorem is related to a result of A. H. Schwarz (see, for instance, [3, p. 39]).

We want to thank S. S. Chern for having suggested this question to us. Thanks are also due to R. Osserman, who read critically a preliminary version of this work, and to R. Gulliver, who pointed out some gaps in our first proof.

Below we present a brief sketch of the proof. A complete proof along with further results will appear elsewhere.

SKETCH OF THE PROOF. Let Δ and K be the laplacian and the gaussian curvature of M, respectively, in the induced metric. Assume that D is not stable. It follows from the Morse index theorem [4] that there

AMS (MOS) subject classifications (1970). Primary 53A10, 49A40, 49F25; Secondary 53B25, 53A30, 58E99, 49F10, 35J99, 35B99.

[1] Partially supported by N.S.F. and C.N.Pq.