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COMPLETE CONVEX HYPERSURFACES OF A HILBERT SPACE

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A *complete convex hypersurface* of a (separable) Hilbert space H is a codimension one C^∞ submanifold of H, which is complete as a metric subspace of H and such that $M=\partial K$, where K is a (closed) convex set with nonvoid interior. For each $p \in M$ let $\nu(p)$ be the unit normal vector which points to the interior of K. In this way we define the *Gauss map* $\nu: M \to \Sigma$ from M into the unit sphere Σ of H. This is a C^∞ map and its derivative at each point $p \in M$ is selfadjoint. We say that *M bounds a half-line* if there exists a half-line $\{p+tv; t \geqq 0\}$ contained in the interior of K. In the finite dimensional case the condition that M bounds a half-line is equivalent to that M is unbounded. In the infinite dimensional case this is not true, as the following simple example shows. Let A be a compact positive definite selfadjoint operator in H and set $M=\{x \in H; \langle A(x), x\rangle = 1\}$. It is not difficult to prove that M is an unbounded positively-curved convex hypersurface and that M does not bound any half-line.

In this note we announce some properties of a complete convex hypersurface M of a Hilbert space. Theorem A characterizes the three possible boundedness situations (bounded, unbounded and bounding a half-line, unbounded and bounding no half-line) in terms of the Gauss map of M. Theorem B gives a necessary and sufficient condition for M to be a pseudograph (see definition below) over one of its tangent hyperplanes. Theorem C is the analogue of the Bonnet-Myers theorem for hypersurface of a Hilbert space. These results are part of my doctoral dissertation. I wish to thank my advisor Professor Manfredo do Carmo for suggesting these problems and for helpful conversations.

THEOREM A. *Let M be a complete convex hypersurface of a Hilbert space H. Then*:

(1) *M is bounded iff the Gauss map $\nu: M \to \Sigma$ is onto.*

(2) *M is unbounded and bounds a half-line iff the image of the Gauss map is contained in a hemisphere.*

(3) *M is unbounded and does not bound any half-line iff the image of the Gauss map is dense and has void interior.*

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