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VECTOR FIELDS GENERATE FEW DIFFEOMORPHISMS

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Let M be a compact C^∞ manifold without boundary. Let $\mathrm{Diff}^r(M)$ be the group of C^r diffeomorphisms of M with the C^r topology. If $r=0$, this corresponds to the group of homeomorphisms of M. A C^s flow on M is a continuous group homomorphism $\varphi: R \to \mathrm{Diff}^s(M)$, $0 \leqq s \leqq \infty$. In a natural way, C^1 vector fields generate C^1 flows and Lipschitz vector fields generate C^0 (topological) flows. We say that $f \in \mathrm{Diff}^r(M)$ embeds in a C^s flow, $s \leqq r$, if f is the map at time one of such a flow.

Our main purpose is to announce results showing that few diffeomorphisms, in the sense of Baire category, embed in flows or are generated by vector fields with some mild differentiability or Lipschitz condition. Here we will prove only one of these results concerning flows generated by vector fields.

Several authors have treated similar questions. For C^2 diffeomorphisms of the circle, our last theorem follows from stronger results of Kopell [2] and it was also proved in [4], where more references can be found. The embedding of diffeomorphisms in topological flows was also considered in [5].

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We now show that with a mild assumption on the vector fields, the diffeomorphisms they possibly generate form a subset of first category in $\mathrm{Diff}^1(M)$.

Fix a riemannian metric on M. Let x be a singularity for a vector field X. X is said to be Lipschitz at x if there exists a constant $K>0$ such that $|X(y)| \leqq K d(x, y)$ for every $y \in M$, where $d(x, y)$ is the distance between x and y. Let χ denote the set of C^0 vector fields on M that generate topological flows and are Lipschitz at the singularities.

THEOREM. *The subset* $F = \{f \in \mathrm{Diff}^1(M) | f = X_{t=1}$ *for some* $X \in \chi\}$ *is of first category in* $\mathrm{Diff}^1(M)$.

PROOF. Consider the set B of Kupka-Smale diffeomorphisms whose periodic points are dense in the nonwandering set. Pugh [6] showed that B is of second category in $\mathrm{Diff}^1(M)$. Following [5], if $f \in B$ and embeds in a topological flow then the periodic points of f are in fact

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