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A SUBSEQUENCE PRINCIPLE IN PROBABILITY THEORY (APPLIED TO THE LAW OF THE ITERATED LOGARITHM)

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It is a remarkable fact that the classical limit laws of probability theory concerning independent identically distributed random variables remain valid, with only minor modifications, for sequences of random variables which are quite far from being independent. This phenomenon has induced me in recent years to formulate the following heuristic principle of subsequences ([2], [3], [4]): let Π be a quantitative asymptotic property valid for any sequence of independent identically distributed random variables X_n belonging to some integrability class defined by a norm $\|\cdot\|_L$; then an analogous property $\tilde{\Pi}$ is valid for a suitable subsequence $\{f_{n(k)}\}$ of *any* sequence of random variables $\{f_n\}$ if only $\sup_n \|f_n\|_L < \infty$. Moreover, the subsequence can be chosen in such a way that any further subsequence of it will have the same property $\tilde{\Pi}$.

When Π is the Kolmogorov strong law of large numbers, the validity of the principle corresponds to a recent theorem of Komlos [8] which states that from any norm-bounded sequence F in L^1 a subsequence F_0 can be so extracted that any arbitrary subsequence $\{f_n\}$ of F_0 has the property that $\lim_{n\to\infty}(f_1+\cdots+f_n)/n=\alpha$ exists a.e. where $\alpha \in L^1$ is the same for all subsequences $\{f_n\}$ of F_0. (Naturally, α depends on the choice of F_0.) When Π is the Marcinkiewicz generalization of the Kolmogorov theorem to independent identically distributed X_n with $\|X_n\|_p<\infty$, $0<p<2$, I have established the validity of the subsequence principle in [1]. For Π of the central limit theorem also, I have shown in [4] in complete detail that the principle in question is perfectly justified. In [2], this and further instances of the principle are discussed. In the present paper my purpose is to announce that the law of the iterated logarithm also falls within the scope of the subsequence principle. More precisely, we have the following theorems.

THEOREM 1. *Let F be any norm-bounded* (*real*) *sequence in L^2 over an arbitrary measure space* (Ω, Σ, μ). *Then there exist functions $\alpha \in L^2$ and*

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