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ON REARRANGEMENTS OF WALSH-FOURIER SERIES AND HARDY-LITTLEWOOD TYPE MAXIMAL INEQUALITIES[1]

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ABSTRACT. In this note we study the a.e. convergence properties of certain rearrangements of the Walsh-Fourier series, and maximal functions of the Hardy-Littlewood type that arise from these rearrangements.

The rearrangements are defined as follows. Let r_n be the nth Rademacher function. For $N=1, 2, \cdots$, let σ_N be a permutation of the nonnegative integers such that $\sigma_N(j)=j$ for all $j\geqq N$. If $2^N\leqq n<2^{N+1}$, $n=\sum_{j=0}^N \varepsilon_j 2^j$, where $\varepsilon_j=0$ or 1 if $0\leqq j\leqq N-1$, and $\varepsilon_N=1$, we define

$$\phi_n = \prod_{j=0}^{N} r_{\sigma_N(j)}^{\varepsilon_j}.$$

We also define $\phi_0=1$ and $\phi_1=r_0$.

If σ_N is the identity permutation, $N=1, 2, \cdots$, we recover the Walsh system. If $\sigma_N(j)=N-j-1$, $0\leqq j\leqq N-1$, $\{\phi_n\}$ is the Walsh-Kaczmarz system. (See [**1**], [**8**] and [**12**].) In general, the system $\{\phi_n\}$ is a rearrangement of the Walsh system within dyadic blocks of indices $2^N\leqq n<2^{N+1}$, $N=1, 2, \cdots$.

We have the following result on the a.e. convergence of Fourier series with respect to $\{\phi_n\}$. For $f\in L^1(0, 1)$, let $S_nf=\sum_{j=0}^{n-1} \phi_j \int_0^1 f\phi_j\, dt$ denote the nth partial sum of the Fourier series of f with respect to $\{\phi_n\}$, and $Mf=\sup_n|S_nf|$.

THEOREM 1. *There are absolute constants C and C_p such that*

(a) $\|Mf\|_p\leqq C_p\|f\|_p, f\in L^p, 2\leqq p<\infty.$

(b) $m\{Mf>y\}\leqq C\exp(-Cy/\|f\|_\infty), y>0, f\in L^\infty.$

This implies the a.e. convergence of S_nf to f for $f\in L^p$, $2\leqq p<\infty$.

If we restrict ourselves to a subclass of rearrangements, we obtain better a.e. convergence results. We say that the permutations $\{\sigma_N\}$ satisfy the

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