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MANIFOLDS OF RIEMANNIAN METRICS WITH PRESCRIBED SCALAR CURVATURE

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1. **Introduction.** Throughout, M will denote a C^∞ compact connected oriented n-manifold, $n \geqq 2$. Let $\rho: M \to \mathbf{R}$ be a C^∞ function, $\mathscr{M}$ the space of C^∞ riemannian metrics on M and

$$\mathscr{M}_\rho = \{g \in \mathscr{M} : R(g) = \rho\}$$

where $R(g)$ is the scalar curvature of g. As in Ebin [**3**], a superscript s will denote objects in the corresponding Sobolev space, $s > n/2+1$ (one can also treat $W^{s,p}$ spaces in the same way), and we also allow $s = \infty$ so $\mathscr{M}^\infty = \mathscr{M}$. Sign conventions on curvatures are as in Lichnerowicz [**10**].

Two of our main results follow:

THEOREM 1. *If ρ is not identically zero or a positive constant, then $\mathscr{M}_s^\rho$ is a smooth submanifold of $\mathscr{M}^s$.*

We can also treat the case $\rho \equiv 0$. Let $\mathscr{F}^s$ denote the set of flat metrics in $\mathscr{M}^s$. Then we have

THEOREM 2. *Assume $\mathscr{F}^s \neq \varnothing$. Writing $\mathscr{M}_0^s = (\mathscr{M}_0^s \backslash \mathscr{F}^s) \cup \mathscr{F}^s$, $\mathscr{M}_0^s$ is the disjoint union of closed submanifolds.*

REMARK. If $\dim M = 2$, $\mathscr{M}_0^s = \mathscr{F}^s$, and if $\dim M = 3$, the hypothesis that $\mathscr{F}^s \neq \varnothing$ can be dropped.

The proof of Theorem 1 also allows us to conclude that a solution h of the linearized equations $DR(g_0) \cdot h = 0$ is tangent to a curve of exact solutions of $R(g) = \rho$ through a given solution g_0, provided ρ is not a constant $\geqq 0$. In the terminology of [**4**] we say the equation $R(g) = \rho$ is *linearization-stable* at g_0. From Theorem 3 below the equation $R(g) = 0$ is still linearization-stable about a solution g_0 provided $\mathrm{Ric}(g_0)$ is not identically zero.

For the singular case $\rho = 0$, Theorem 2 incorporates an isolation theorem inspired by the work of Brill and Deser [**2**], namely, that the flat metrics are isolated solutions of $R(g) = 0$. As a corollary one has: *If $g(t)$ is a*

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