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ON THE INCOMPATIBILITY OF TWO CONJECTURES CONCERNING PRIMES; A DISCUSSION OF THE USE OF COMPUTERS IN ATTACKING A THEORETICAL PROBLEM[1,2]

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Introduction. This talk is about the interplay between computers and theoretical research, as experienced by someone who is not a computer expert. The story involves, among other things, a measure of good luck. Several instances of this will emerge in due course, but one example now may give the idea: The speaker and his co-worker, Douglas Hensley, used a computer to seek a certain combinatorial pattern. Our first attempts failed; however the desired patterns did exist, and we eventually found infinitely many of them by theoretical means. If, on the first day, the computer had given us the result we wanted, we probably would have stopped there and missed the further developments.

In line with the purpose of this talk, I intend to be somewhat informal and omit certain details. The main theme is the narrative, which relates how a theoretical argument emerged from a computer search. This begins in Part II, using definitions given in Part I, and the argument itself is sketched at the end of the talk. (A detailed proof of our results will appear in [5].)

As I have mentioned, this work was a collaboration with Douglas Hensley. We were aided in an essential way by William Franta and Richard Franta of our computer sciences department. They programmed a CDC 6400 computer to handle sieving operations on 100,000 points. We used a time-sharing circuit, which proved very helpful since it provided us with instant reinforcement, positive or negative—mostly negative as it turned out.

Our objective was to seek a counterexample to a conjecture. The conjecture involves two functions, the familar prime-counting function $\pi(x)$, and a second function $\rho^*(x)$ related to sieves, which we will define presently. For small values of x, one finds that $\rho^*(x)$ is always smaller than $\pi(x)$, and this inequality was believed to hold generally. Furthermore,

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The written version of an invited address given at the Summer Meeting in Missoula, Montana on August 23, 1973. It is not a verbatim rendering of the talk, but is similar in spirit; received by the editors October 1, 1973.

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