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ADDENDUM TO: "ON EXTENSIONS OF FUNDAMENTAL GROUPS OF SURFACES AND RELATED GROUPS"

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Copying the methods of J. Nielsen [**1**] Theorem 1 of [**2**] can be proved, i.e. that a finite torsionfree extension of the fundamental group of a surface is isomorphic to the fundamental group of a surface. Indeed, the following slightly more general theorem can be proved, but it is considerably weaker than Theorem 1′ of [**2**].

THEOREM. *Let $\mathfrak{F}$ be the fundamental group of a surface S and let $\mathfrak{F}$ be finitely generated. Let $\mathfrak{G}$ be a group which contains $\mathfrak{F}$ as a normal subgroup of finite index and which has the following properties:*

(i) *For each $g \in \mathfrak{G}$ the automorphism of $\mathfrak{F}$ defined by $x \mapsto g^{-1}xg$ is induced by a homeomorphism of S.*

(ii) *If $g \in \mathfrak{G}$ and $g^{-1}xg = x$ holds for all $x \in \mathfrak{F}$, then $g \in \mathfrak{F}$.*

(iii) *If $x^a = y^b = (xy)^c = 1$ holds for $x, y \in \mathfrak{G}$ and $a, b, c \geqq 2$, then x, y generate a cyclic subgroup of $\mathfrak{G}$.*

Then $\mathfrak{G}$ is isomorphic to a finitely generated discontinuous group of motions of the hyperbolic or euclidean plane.

I shall briefly sketch a proof of the Theorem which generalizes [**1**]. Let S be an orientable surface with finite genus and a finite number of holes and without boundary. We consider S as a Riemann surface. If the universal cover is holomorphically equivalent to the euclidean plane, everything can be proved in a similar way as in [**2**, Theorem 3]. Therefore we may assume that the universal cover is the hyperbolic plane $\boldsymbol{H}$ which we represent by the unit disk $\{z \in \boldsymbol{C} \mid |z| < 1\}$ and the Poincaré model. The fundamental group of S acts on $\boldsymbol{H}$ as a group $\mathfrak{F}$ of conformal transformations. We may assume that $\mathfrak{F}$ contains only hyperbolic transformations except the identity. Then the methods of [**1**] can be applied: Each cyclic subgroup of $\mathfrak{F}$ consists of motions with the same axis, and a maximal cyclic subgroup contains all elements preserving an axis. Therefore each automorphism of $\mathfrak{F}$ induces a permutation of the axes of $\mathfrak{F}$ and

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