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MINIMAL TOTAL ABSOLUTE CURVATURE FOR ORIENTABLE SURFACES WITH BOUNDARY

BY JAMES H. WHITE

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Let M be an orientable surface with single smooth boundary curve C which is C^2 imbedded in Euclidean three-space E^3. (M may be thought of as a closed orientable surface with a single disc removed.) Let M_ε be the set of points of E^3 at a distance ε from M. M_ε is, of course, for small ε, an imbedded closed surface which is almost everywhere C^2. Using N. Grossman's [**1**] adaptation of N. Kuiper's [**2**] definition, we say that M has minimal total absolute curvature if M_ε is tightly imbedded or has the two piece property, TPP [**2**].

We announce the following result:

THEOREM. *Let M be an orientable surface of genus g with a single smooth boundary curve which is C^2 imbedded in E^3. Then M has minimal total absolute curvature if and only if M has $g=0$ and is a planar disc bounded by a convex curve.*

The proof uses a series of integral equations and geometric arguments. The outline is as follows. First, in his paper [**1**], N. Grossman shows that an orientable surface M of genus g with boundary curve C has minimal total absolute curvature only if the following integral equality holds:

$$(1) \qquad \frac{1}{2\pi}\int_M |K|\, dA + \frac{1}{2\pi}\int_C \kappa\, ds = 1 + 2g,$$

where K is the Gauss curvature of M and κ is the Frenet curvature of the boundary curve C considered as a space curve in E^3, where dA is the area element of M and ds is the arc element of C. Note that the right-hand side is the sum of the betti-numbers of M and compare with Kuiper [**2**] for closed surfaces.

Next, the theorem of Gauss-Bonnet yields

$$(2) \qquad \frac{1}{2\pi}\int_M K\, dA + \frac{1}{2\pi}\int_C \kappa_g\, ds = 1 - 2g,$$

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