

ASYMPTOTIC STABILITY WITH RATE CONDITIONS FOR DYNAMICAL SYSTEMS

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We announce existence and smoothness theorems for invariant manifolds characterized by asymptotic stability at specified rates. Our theorems require roughly half of a hyperbolic structure, so various known results about stable and unstable manifolds are included as special cases. Even under hyperbolicity assumptions, however, our results give new information about metric properties, invariant foliations, and asymptotic stability with asymptotic phase. Proofs will appear in [2].

1. Invariant sets with expanding structure. Let U and V be open subsets of a C^r manifold M , $1 \leq r \leq \infty$, and let $F: U \rightarrow V$ be a C^r diffeomorphism. $\Lambda \subset U$ is called weakly overflowing invariant (under F) if $\Lambda \subset F(\Lambda)$. Let $TF: TU \rightarrow TV$ be the map induced by F on tangent spaces. A subbundle $E \subset TM|_{\Lambda}$ is called weakly overflowing invariant if $E \subset TF(E)$. We assume that all bundles have constant dimension.

Suppose that Λ is compact, and that Λ and E are weakly overflowing invariant. Choose any vector bundle $N \subset TM|_{\Lambda}$, complementary to E , and let π^E and π^N be the projections corresponding to the splitting $TM|_{\Lambda} = E \oplus N$. For any $m \in \Lambda$, and any $v^0 \in E_m$, $w^0 \in N_m$, let

$$v^{-k} = DF^{-k}(m)v^0, \quad w^{-k} = \pi^N DF^{-k}(m)w^0,$$

where $DF^{-k}(m): TM_m \rightarrow TM_{F^{-k}(m)}$ is the map induced by F^{-k} .

Choose a Riemannian metric for TM , and let $|\cdot|$ be the corresponding norm. Define $\alpha^*(m) = \inf\{\alpha > 0: |v^{-k}|/\alpha^k \rightarrow 0 \text{ as } k \rightarrow \infty \text{ for all } v^0 \in E_m\}$, $\rho^*(m) = \inf\{\rho > 0: [|v^{-k}|/|w^{-k}|]/\rho^k \rightarrow 0 \text{ as } k \rightarrow \infty \text{ for all } v^0 \in E_m, w^0 \in N_m\}$. The pair (Λ, E) is called an invariant set with expanding structure if $\alpha^*(m) < 1$, $\rho^*(m) < 1$ for all $m \in \Lambda$. This definition follows the form developed in [1].

LEMMA. α^* and ρ^* are constant on orbits and do not depend on the choice of N or the metric for TM .

LOCAL EXPANDING FAMILY THEOREM. *Let (Λ, E) be an invariant set with expanding structure. Then there is a family of compact manifolds*

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