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COEFFICIENTS FOR ALPHA-CONVEX UNIVALENT FUNCTIONS

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Let α be a nonnegative real number, and let $M(\alpha)$ denote the class of normalized α-convex univalent functions f in the open unit disc $E=\{z:|z|<1\}$, i.e., $f\in M(\alpha)$ if and only if f is regular in E, $f(0)=f'(0)-1=0$, $f(z)f'(z)/z\neq 0$ for $z\in E$, and

$$\operatorname{Re}\left\{(1-\alpha)\frac{zf'(z)}{f(z)}+\alpha\left[1+\frac{zf''(z)}{f'(z)}\right]\right\}>0$$

for $z\in E$ [3], [4]. If $f(z)=z+\sum_{n=2}^{\infty}a_nz^n$, the coefficient bounds for $|a_2|$ and $|a_3|$ are known [2], [4]; an inequality relating the coefficients $|a_n|$ for $n=2,3,\cdots$ is found in [2]; yet the determination of the coefficient bound for $|a_n|$ has so far been an open problem.

Here we announce the general result for this coefficient problem; the proof will be published elsewhere.

THEOREM. *Let* $f(z)=z+\sum_{k=2}^{\infty}a_kz^k\in M(\alpha)$. *Let* $S(n)$ *be the set of all* n*-tuples* $(r_1,r_2,\cdots,r_n)$ *of nonnegative integers for which* $r_1+2r_2+3r_3+\cdots+nr_n=n$, *and for each such* n*-tuple define* m *by* $r_1+r_2+\cdots+r_n=m$. *If* $\gamma(\alpha,m)=\alpha(\alpha-1)(\alpha-2)\cdots(\alpha-m)$ *with* $\gamma(\alpha,0)=1$, *then for* $n=1,2,\cdots$

$$|a_{n+1}|\leqq\sum\frac{\gamma(\alpha,m-1)c_1^{r_1}c_2^{r_2}\cdots c_n^{r_n}}{r!\,r_2!\cdots r_n!},\tag{1}$$

where summation is taken over all n*-tuples in* $S(n)$, *and*

$$c_n=\frac{2(2+\alpha)(2+2\alpha)\cdots[2+(n-1)\alpha]}{n!\,\alpha^n(1+n\alpha)}.$$

The bounds in (1) are sharp and for $\alpha>0$ attained by

$$f(z)=\left[\frac{1}{\alpha}\int_0^z\zeta^{1/\alpha-1}(1-\zeta)^{-2/\alpha}\,d\zeta\right]^{\alpha}.$$

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