BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 80, Number 2, March 1974

LATERAL COMPLETION FOR ARBITRARY LATTICE GROUPS

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Communicated by Joseph Rotman, August 2, 1973

1. **Introduction.** An old problem in the theory of lattice groups is to construct, for a given lattice group G, a canonical extension, $\hat{G}$, with the property that if M is a subset of $\hat{G}$ such that $x \wedge y=0$ if $x, y \in M$ and $x \neq y$, then M has a supremum in $\hat{G}$. The problem was first solved, for conditionally complete vector lattices, by Nakano [7] and was also treated by Pinsker [8] with the conditional completeness assumption.

The property required of $\hat{G}$ above is called *lateral completeness*. The requirement that G should be canonical is met by defining the *lateral completion* of G to be a laterally complete extension $\hat{G}$ of G such that: (i) no sublattice group of $\hat{G}$ is laterally complete and contains G; and (ii) G is dense in $\hat{G}$, which means that if $f \in \hat{G}$ and $f > 0$, there exists $g \in G$ such that $0 < g \leqq f$. If requirement (ii) is dropped we call $\hat{G}$ an $\mathscr{L}$-completion of G.

The existence and uniqueness of a lateral completion of G are consequences of the orthocompletion of the author [1] in the case G is representable. Conrad [5] simplified the proofs in [1] and also showed the existence and uniqueness of a lateral completion if G has zero radical. Conrad's paper points out that $\mathscr{L}$-completions exist in profusion, as a consequence of the Holland representation [6], and that they can be highly pathological. Byrd and Lloyd [4] showed that Conrad's methods can be pushed through to the completely distributive case.

2. **The main theorem.** We have the following

THEOREM. *Every lattice group G has a lateral completion $\hat{G}$ which is unique up to isomorphism.*

The proof of this theorem is long and technical. We outline the method.

Let $\mathscr{D}$ be the set of pairwise disjoint subsets of the lattice group G; i.e. $M \in \mathscr{D}$ if $\varnothing \neq M \subset G$ and $x \wedge y=0$ for $x, y \in M$, $x \neq y$. We first construct

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AMS (MOS) subject classifications (1970). Primary 06A55; Secondary 06A50, 06A60, 06A75, 06A35.

Key words and phrases. Lateral completion, orthocompletion, radical of a lattice group, distributive radical of a lattice group, representable lattice group.

[1] Preparation of this paper was partially supported by NSF grant GP27916.