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ORBIT STRUCTURE OF THE EXCEPTIONAL HERMITIAN SYMMETRIC SPACES. I

BY DANIEL DRUCKER

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In this note, we use an algebraic construction of J. Tits [7], [8] to obtain results on the orbit structure of the exceptional hermitian symmetric spaces. These results complete the explicit analysis of the orbit structure of hermitian symmetric spaces that was given by J. A. Wolf [9, pp. 321–356] for the classical cases only.

Part I is concerned with the space $E_7/E_6 \cdot SO(2)$. Part II will treat the other exceptional space, $E_6/SO(10) \cdot SO(2)$. Full details and complete proofs will appear in a longer article.

1. **J. Tits' construction of the complex Lie algebra $\mathfrak{E}_7$.** Let $\mathscr{A}$ be the algebra of 2×2 matrices with entries in $\boldsymbol{C}$ and let $\mathscr{J}$ be the 27-dimensional Jordan algebra of hermitian 3×3 matrices whose entries are complex Cayley numbers. Let $\mathscr{A}_0$ and $\mathscr{J}_0$ be the subsets of $\mathscr{A}$ and $\mathscr{J}$ consisting of matrices with zero trace. Also let $\operatorname{Der}(\mathscr{J})$ be the Lie algebra of derivations of $\mathscr{J}$. Let $\{L(A)\}(B)=A \circ B$ denote left multiplication by A in $\mathscr{J}$, and let $[a, b]=ab-ba$ for $a, b \in A$. Now define a bilinear, anticommutative multiplication [,] on the complex vector space

$$\mathfrak{g} = (\mathscr{A}_0 \otimes \mathscr{J}) + \operatorname{Der}(\mathscr{J}) \tag{1}$$

by means of the following rules:

(a) $[D, D']$ is the usual commutator for $D, D' \in \operatorname{Der}(\mathscr{J})$.

(b) $[D, a\otimes A]=a\otimes D(A)$ for $a \in \mathscr{A}_0$, $A \in \mathscr{J}$, and $D \in \operatorname{Der}(\mathscr{J})$.

(c) $[a\otimes A, b\otimes B]=\frac{1}{2}[a, b]\otimes A \circ B+\frac{1}{2}\operatorname{Tr}(ab)[L(A), L(B)]$ for $a, b \in \mathscr{A}_0$ and $A, B \in \mathscr{J}$.

It is a theorem of J. Tits that $\mathfrak{g}$ is the complex Lie algebra $\mathfrak{E}_7$.

Let $\mathscr{A}'$ be the set of matrices in $\mathscr{A}$ with real entries and $\mathscr{A}''$ the set of matrices in $\mathscr{A}$ of the form $\left[\begin{smallmatrix} u & v \\ -v^* & u^* \end{smallmatrix}\right]$, where $u, v \in \boldsymbol{C}$ and where the asterisks indicate complex conjugation. Let $\mathscr{J}'$ be the set of matrices in $\mathscr{J}$ whose entries are real Cayley numbers. If we substitute $\mathscr{A}'$ and $\mathscr{J}'$

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