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A WEIGHTED NORM INEQUALITY FOR FOURIER SERIES

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Let $Mf(x)=\sup|S_nf(x)|$, where S_nf denotes the nth partial sum of the Fourier series of f. We will show

$$(1) \qquad w \in A_p,\ p > 1, \quad \text{implies} \quad \int [Mf]^p w \leqq C \int |f|^p\, w.$$

Recall that a nonnegative weight function $w \in A_p$, $p>1$, if there is a constant K such that

$$\left(\int_I w\right)\left(\int_I w^{-1/(p-1)}\right)^{p-1} \leqq K\,|I|^p$$

for all intervals I. The A_p condition, $p>1$, characterizes all weights w for which the mapping of f into the Hardy-Littlewood maximal function of f is bounded on the weighted L^p space $L^p(w)$. (See Muckenhoupt [**6**].) This fundamental fact leads to boundedness on $L^p(w)$ for other operators which can be associated with the Hardy-Littlewood maximal function. For example, the conjugate function and more general singular integrals are of this type. Also, $\int |S_nf-f|^p w \to 0$ $(n\to\infty)$ if and only if $w \in A_p$, $p>1$. (See Hunt, Muckenhoupt and Wheeden [**5**] and Coifman [**3**].) It follows that the inequality in (1) holds only if $w \in A_p$, $p>1$.

Our proof of (1) follows closely the proof in Coifman [**3**]. We will prove a Burkholder-Gundy type distribution function inequality which relates the weighted distribution functions of modified versions of Mf and the Hardy-Littlewood maximal function of f. (See Burkholder and Gundy [**1**].) To do this we will use the boundedness of M on L^r, $r>1$, and an extremely useful consequence of the A_p condition which relates the w-weighted measure and the Lebesgue measure of certain types of sets. This useful property is closely related to the development of Muckenhoupt [**6**] and was first explicitly used in connection with a distribution

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