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COVERING AND FUNCTION THEORETIC PROPERTIES OF UNIFORM SPACES

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The purpose of this note is to announce the major ideas and results developed in [**R**]$_1$. The proofs of these results will appear in a series of three papers [**R**]$_2$, [**R**]$_3$, and [**RR**], the latter including categorical topics that will be omitted here. The subject matter is the covering and function theoretic properties of uniform spaces, a subject initiated by John Isbell in the 1950's. (See [**GI**] and [**I**].) Our work represents a continuation and extension of the current work of Anthony Hager ([**H**]$_1$, [**H**]$_2$) and Z. Frolík; and overlaps somewhat with recent work of Z. Frolík ([**Fr**]$_1$, [**Fr**]$_2$). The author wishes to emphasize that his work substantiates the existence of a theory of uniform structures *which is not primarily* interested in topological applications. Therefore, the viewpoint adopted here is one of intrinsic interest per se in uniform properties.

A uniform space is denoted by uX, where u is a family of covers on the set X constituting a uniformity. uX is *fine* if u is the largest uniformity on X with the same uniform topology. A subfine space is a subspace of a fine space. uX is *locally fine* if each cover of the form $\{A_\alpha \cap C^\alpha_\beta\} \in u$, where $\{A_\alpha\} \in u$, and $\{C^\alpha_\beta\} \in u$ for each α. uX is *M-fine* (*sub-M-fine*) if each uniformly continuous function (map) to a metric (complete metric) space remains a map relative to the fine uniformity on M (the uniformity with the basis of open covers of M). uX is *hereditarily M-fine* if each subspace is M-fine.

The basic source on locally fine and subfine spaces is [**I**], while the development of separable M-fine and separable hereditarily M-fine spaces (those with a basis of countable covers) originates in [**H**]$_1$ and [**H**]$_2$.

One easily sees that each fine space is M-fine and that each M-fine space is sub-M-fine. Example C of [**GI**] is a hereditarily M-fine space which is not locally fine. [**I**] shows that each locally fine space is sub-M-fine and that each subfine space is locally fine; the converse of the latter is an unsolved problem. From [**I**] we also know that each separable sub-M-fine space is subfine.

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