

SELF-ORTHOGONAL LATIN SQUARES OF ALL ORDERS $n \neq 2, 3, 6$

BY R. K. BRAYTON, DONALD COPPERSMITH AND A. J. HOFFMAN¹

Communicated by Gian-Carlo Rota, July 10, 1973

1. Introduction. E. Nemeth [15] has coined the term “self-orthogonal latin square” to denote a latin square orthogonal to its transpose. We have proved that there exists a self-orthogonal latin square of order n if and only if $n \neq 2, 3, 6$. Previous literature on this problem ([3], [6]–[10], [12]–[15], [17]–[19]) had constructed such squares for certain infinite classes of n and certain isolated values. We only became aware of this work after having constructed s.o.l.s. for all but approximately a dozen values of n , for we were motivated to consider the question by a problem in tennis. This we describe in §2. In §3 we give a rough summary of the proof of the theorem, details of which will appear elsewhere.

2. Spouse-avoiding mixed doubles round robins. In tennis, a mixed doubles match consists of two teams, each of which consist of one man and one woman. In informal play at a tennis club, it is common for a husband and wife to be a team, but this is not always so. One of us was asked by John Melian [11], director of the Briarcliff Racquet Club in Briarcliff, New York, to arrange a schedule of matches for n couples in which husband and wife did not necessarily play together. We interpreted this as follows:

(i) Husband and wife would *never* appear in the same match either as partners or opponents.

(ii) Each pair of players of the same sex would oppose each other exactly once.

(iii) Each pair of players of opposite sex, if not married to each other, would play in exactly one match as partners, and exactly one match as opponents.

Suppose we are given a s.o.l.s. A of order n . It follows that the diagonal entries are a rearrangement of $\{1, \dots, n\}$, and without loss of generality we may assume $a_{ii} = i$. We now determine the $\binom{n}{2}$ matches in a spouse-

AMS (MOS) subject classifications (1970). Primary 05B15.

¹ The work of this author was supported (in part) by the U.S. Army under contract #DAHC04-72-C-0023.