PERSISTENT MANIFOLDS ARE NORMALLY HYPERBOLIC

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Let $M$ be a compact, smooth, boundaryless manifold, and let $\text{Diff}^1(M)$ be the group of $C^1$-diffeomorphisms of $M$ endowed with the $C^1$ topology. If $f \in \text{Diff}^1(M)$, a compact invariant manifold of $f$, $V$, is a $C^1$ boundaryless submanifold of $M$, such that $f(V) = V$. In [1] and [2], M. Hirsch, C. Pugh and M. Shub proved that a normally hyperbolic compact invariant manifold (Definition 1 below) is persistent (Definition 2 below). The main purpose of this note is to announce the proof of the converse of this theorem.

1. The main theorem.

DEFINITION 1. If $f \in \text{Diff}^1(M)$, a compact invariant manifold of $f$, $V$, is normally hyperbolic if the tangent bundle of $M$ restricted to $V$, $TM|_V$, has a splitting $TM|_V = TV \oplus N^s V \oplus N^u V$ where $TV$ is the tangent bundle of $V$, and $N^s V$, $N^u V$, are $Tf$-invariant subbundles of $TM|_V$ such that there exist constants $K > 0$, $0 < \lambda < 1$ satisfying

$$\|(Tf)^n_x(N^s V)_x\| \leq K\lambda^n, \quad (Tf)^{-n}_x(N^s V)_x \leq K\lambda^n,$$

$$\|(Tf)^n_x(N^u V)_x\| \leq K\lambda^n, \quad (Tf)^{-n}_x(N^u V)_x \leq K\lambda^n,$$

for all $x \in V$, $n \in \mathbb{Z}^+$.

DEFINITION 2. If $f \in \text{Diff}^1(M)$, a compact invariant manifold of $f$, $V$, is persistent if there exist a neighborhood $\mathcal{U}$ of $f$ in $\text{Diff}^1(M)$ and a neighborhood $U$ of $V$ in $M$ such that:

(a) For all $g \in \mathcal{U}$, $V_g = \bigcap_{n \in \mathbb{Z}} g^*(U)$ is a $C^1$ boundaryless submanifold of $M$ and $V_f = V$.

(b) If $g$ is $C^1$ near to $f$, $V_g$ is $C^1$ near to $V_f$.


1 The results announced here are part of the author's doctoral thesis at IMPA under the guidance of J. Palis.