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PERSISTENT MANIFOLDS ARE NORMALLY HYPERBOLIC

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Let M be a compact, smooth, boundaryless manifold, and let $\mathrm{Diff}^1(M)$ be the group of C^1-diffeomorphisms of M endowed with the C^1 topology. If $f \in \mathrm{Diff}^1(M)$, a compact invariant manifold of f, V, is a C^1 boundaryless submanifold of M, such that $f(V)=V$. In **[1]** and **[2]**, M. Hirsch, C. Pugh and M. Shub proved that a normally hyperbolic compact invariant manifold (Definition 1 below) is persistent (Definition 2 below). The main purpose of this note is to announce the proof of the converse of this theorem.

1. The main theorem.

DEFINITION 1. If $f \in \mathrm{Diff}^1(M)$, a compact invariant manifold of f, V, is normally hyperbolic if the tangent bundle of M restricted to V, TM/V, has a splitting $TM/V = TV \oplus N^sV \oplus N^uV$ where TV is the tangent bundle of V, and N^sV, N^uV, are Tf-invariant subbundles of TM/V such that there exist constants $K>0$, $0<\lambda<1$ satisfying

$$\|(Tf)_x^n/(N^sV)_x\| \leqq K\lambda^n,$$

$$\|(Tf)_x^{-n}/(N^uV)_x\| \leqq K\lambda^n,$$

$$\|(Tf)_x^n/(N^sV)_x\| \, \|(Tf)_{f^n(x)}^{-n}/(TV)_{f^n(x)}\| \leqq K\lambda^n,$$

$$\|(Tf)_x^{-n}/(N^uV)_x\| \, \|(Tf)_{f^{-n}(x)}^{n}/(TV)_{f^{-n}(x)}\| \leqq K\lambda^n,$$

for all $x \in V$, $n \in Z^+$.

DEFINITION 2. If $f \in \mathrm{Diff}^1(M)$, a compact invariant manifold of f, V, is persistent if there exist a neighborhood $\mathcal{U}$ of f in $\mathrm{Diff}^1(M)$ and a neighborhood U of V in M such that:

(a) For all $g \in \mathcal{U}$, $V_g = \bigcap_{n\in Z} g^n(U)$ is a C^1 boundaryless submanifold of M and $V_f = V$.

(b) If g is C^1 near to f, V_g is C^1 near to $V_f = V$.

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[1] The results announced here are part of the author's doctoral thesis at IMPA under the guidance of J. Palis.