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STABILITY AND TRANSVERSALITY

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1. **Introduction.** Let N and P be C^∞ manifolds of dimensions n and p and let $C^\infty(N, P)$ denote the space of all C^∞ mappings $f: N \to P$ with the fine C^∞ topology [**2**, II, p. 259]. A mapping $f \in C^\infty(N, P)$ may be stable in either the C^∞ [**2**, II] or topological [**3**] sense. In this paper we state certain results connecting these two concepts of stability. In a related development we also outline a procedure for showing that topologically stable mappings satisfy certain transversality conditions. All of the results given here are based on our thesis [**4**] to which we refer for proofs and further details.

2. **A conjecture.** It is clear that any C^∞ stable mapping is also topologically stable, but the converse is false in general. In fact for N compact Mather has shown that the topologically stable mappings are always dense in $C^\infty(N, P)$ [**3**], while the C^∞ stable mappings are dense if and only if n, p lie in a certain "nice" range [**2**, VI]. However, one may still conjecture the following:

(2.1) *If N is compact and n, p lie in the "nice" range, then any topologically stable mapping*

$$f: N \to P$$

is also C^∞ stable.

In [**4**] we verify the above conjecture for the comparatively simple cases $p > 2n$ ("Whitney embedding" range) and $p = 1$ ("functions"). We obtain related results for a more substantial range of dimensions by introducing a "uniform stability" condition.

DEFINITION. $f \in C^\infty(N, P)$ is *uniformly stable* provided that for any family

$$F: (\boldsymbol{R}^K, 0) \to (C^\infty(N, P), f)$$

of maps (parameterized by $\boldsymbol{R}^K$, any $K > 0$) for which the associated map

$$\tilde{F}: N \times \boldsymbol{R}^K \to P \times \boldsymbol{R}^K$$

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