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A STRUCTURE THEORY OF JORDAN PAIRS

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1. **Definitions.** Let V^σ be modules over a ring k where $\sigma = \pm$. For a quadratic map $Q^\sigma: V^\sigma \to \mathrm{Hom}_k(V^{-\sigma}, V^\sigma)$ let

$$L^\sigma(x, y)z = Q^\sigma(x+z)y - Q^\sigma(x)y - Q^\sigma(z)y = \{xyz\}.$$

A *Jordan pair* over k is a pair $\mathscr{V} = (V^+, V^-)$ of k-modules together with a pair (Q^+, Q^-) of quadratic maps $Q^\sigma: V^\sigma \to \mathrm{Hom}_k(V^{-\sigma}, V^\sigma)$ such that the identities

(JP1) $$L^\sigma(x, y)Q^\sigma(x) = Q^\sigma(x)L^{-\sigma}(y, x),$$

(JP2) $$L^\sigma(Q^\sigma(x)y, y) = L^\sigma(x, Q^{-\sigma}(y)x),$$

(JP3) $$Q^\sigma(Q^\sigma(x)y) = Q^\sigma(x)Q^{-\sigma}(y)Q^\sigma(x),$$

hold in all base ring extensions. Jordan pairs have first been studied by K. Meyberg in [**6**], although not in the present form.

A *homomorphism* $h: \mathscr{V} \to \mathscr{W}$ of Jordan pairs is a pair $h = (h^+, h^-)$ of k-linear maps $h^\sigma: V^\sigma \to W^\sigma$ such that $h^\sigma Q^\sigma(x) = Q^\sigma(h^\sigma(x))h^{-\sigma}$, for all $x \in V^\sigma$. The *opposite* of $\mathscr{V}$ is $\mathscr{V}^{\mathrm{op}} = (V^-, V^+)$ with quadratic maps (Q^-, Q^+). An *antihomomorphism* from $\mathscr{V}$ to $\mathscr{W}$ is a homomorphism from $\mathscr{V}$ to $\mathscr{W}^{\mathrm{op}}$. An antiautomorphism η of $\mathscr{V}$ is called an *involution* if $\eta^{-\sigma}\eta^\sigma$ is the identity on V^σ.

2. **Connections with Jordan algebras and Jordan triple systems.** There is a one-to-one correspondence between Jordan triple systems (cf. [**7**]) and Jordan pairs with involution as follows: If η is an involution of the Jordan pair $\mathscr{V}$ then V^+ becomes a Jordan triple system with quadratic operators $P(x) = Q^+(x)\eta^+$. If conversely (V, P) is a Jordan triple system then (V, V) is a Jordan pair with $Q^\sigma(x)y = P(x)y$ and involution $\eta^\sigma = \mathrm{Id}_V$. The structure group of the Jordan triple system is the automorphism group of the corresponding Jordan pair.

Let $\mathscr{V}$ be a Jordan pair. An element $a \in V^+$ is called invertible if $Q^+(a)$ is invertible. There is a one-to-one correspondence between Jordan pairs containing invertible elements and isotopism classes of unital quadratic

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