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MOTIONS OF LINKS IN THE 3-SPHERE

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1. **Introduction.** A motion in M of a subspace N consists of an (ambient) isotopy of N through M which ultimately returns N to itself. Here we study the problem of determining all essentially different motions and the natural group structure on this set which is induced when two motions are multiplied by performing them on N in succession.

The aim of this paper is to calculate the group of motions of links in the 3-sphere and in 3-space. In §3 this is reduced, for "links with generalized axis", to a calculation of isotopy classes of homeomorphisms of a surface punctured in a finite number of points. In §4, generators are given for the motion group of a torus link in S^3, and generators and relations are given for the motion group of a torus knot in $\boldsymbol{R}^3$ (in fact I can give generators and relations for both groups).

We begin with a definition of motion groups based on Dahm's original definition:

2. **Motion groups.** Let M be a manifold, N a subspace contained in the interior of M. Denote by $H(M)$ the group of autohomeomorphisms of M with the compact open topology, where if M has boundary ∂M, all homeomorphisms are required to fix ∂M pointwise; and let $H(M; N)$ be the subgroup of maps restricting to an autohomeomorphism of N, with the subspace topology. The notation $\mathscr{H}(M)$ and $\mathscr{H}(M; N)$ is used for the group of path components of $H(M)$ and $H(M; N)$, respectively. Denote the identity map of M by $1_M: M \to M$.

A *motion of N in M* is a path f_t in $H(M)$ beginning at $f_0 = 1_M$ and ending at f_1 where $f_1(N) = N$. The motion f is said to be a *stationary motion of N in M* if $\forall t, f_t(N) = N$. To compose two motions, translate the second by multiplication in the group $H(M)$ so that its initial endpoint coincides with the terminal endpoint of the first, and multiply as in the groupoid of paths. Define the *inverse f^{-1} of a motion f* to be the inverse path of the path f in $H(M)$, translated so that its initial endpoint is 1_M. Finally, we say that the motions f and g are equivalent if the path $g^{-1} \circ f$ is homotopic modulo its endpoints to a stationary motion. Thus stationary motions of N in M are to be considered trivial; i.e., they are equivalent to the trivial path mapping to the point 1_M. The *group of motions of N in M*, denoted

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