BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 80, Number 1, January 1974

A DEGREE FOR NONACYCLIC MULTIPLE-VALUED TRANSFORMATIONS

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Communicated by Mary Ellen Rudin, July 2, 1973

The objective of this note is the definition of a degree generalizing that of Leray-Schauder [**1**] for certain possibly nonacyclic set-valued compact transformations of a ball in an arbitrary Banach space. The degree definition achieved extends that of [**3**] and depends on the material in [**2**], [**3**], [**4**] and [**5**]. It is new even for the finite-dimensional case.

Let E be a Banach space. Write D for the closure of a convex open set and $\dot{D}$ for its boundary. Denote by E_N an N-dimensional subspace of E and by D_N and $\dot{D}_N$ the intersections $E_N \cap D$ and $E_N \cap \dot{D}$, respectively. (We tacitly assume E is infinite dimensional but the finite-dimensional consequences amount to identification of E and some E_N.)

The symbol F refers to a transformation $F: D \to E$ which is upper semicontinuous, takes points into closed sets, and is compact in the sense that $\mathrm{cl}(F(D))$ is compact. Finally, for every closed set K, $F^{-1}(K)$ is required to be closed. Among other consequences, the graph $\Gamma(F)$ is closed where $\Gamma(F) = \{(x, y) \mid y \in F(x),\ x \in D\} \subset D \times C,\ C$ compact$\}$. We write $\Gamma(F_N)$ and $\Gamma(\dot{F}_N)$ for the corresponding graphs when D is replaced by D_N or $\dot{D}_N$. A *fixed point* $\bar{x}$ of F satisfies $\bar{x} \in F(\bar{x})$.

Denote the *r-dimensional singular set* by $\sigma_r = \{x \mid H^r F(x) \neq 0\}$ where the cohomology groups are assumed to be the Alexander Spanier reduced groups with integer coefficients. The total singular set is denoted by $\sigma = \bigcup_r \sigma_r$.

Let p denote the effective bound for nonacyclicity, namely

$$p = 1 + \sup_r \{r + \dim \sigma_r \mid \sigma_r \neq \varnothing\},$$

where $\dim \sigma_r$ is the maximum covering dimension for finite covers of subsets of σ_r which are closed in D.

The transformation F is *admissible* if, besides the earlier restrictions, (a) F is fixed point free on $\dot{D}$, (b) $\sigma_r = \varnothing$ except for a finite set of indices, (c) σ_r is contained in a finite subspace E_S, and (d) for $x \in \sigma$, $H^* F(x)$ is

AMS (MOS) subject classifications (1970). Primary 47H10, 54C60, 54H25.

Key words and phrases. Set valued transformations, Leray-Schauder degree, Banach space, Alexander cohomology.