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A CONTINUITY PROPERTY AND SURFACE TOPOLOGY

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Introduction. While studying global properties of spaces of retractions of 2-manifolds, the author was led to a continuity property for conformal mappings of simply connected domains with locally connected boundaries (§1, Theorem 2). Applications are given to the study of spaces of retractions (§2), spaces of retracts (§3), and the Dirichlet problem (§4). Details of the proofs will appear elsewhere.

1. **The continuity property.** For each nonnegative integer i, let G_i be a bounded simply connected domain in the plane E^2 such that each G_i contains some fixed closed disk D with the origin as center. We first state a known result.

THEOREM 1 (CARATHÉODORY, ET AL.; BORSUK). *For each i, the following are equivalent.*

(a) *The boundary F_i of G_i is locally connected.*

(b) *There is a unique continuous surjection f_i from the closed unit disk B^2 to* $\mathrm{cl}(G_i) = G_i \cup F_i$ *which fixes the origin, is conformal on* $\mathrm{int}(B^2)$, *and has positive derivative at the origin.*

(c) *There is a retraction of* $E^2\backslash\mathrm{int}(D)$ *with image* $E^2\backslash G_i$.

From [**8**, p. 112] it is clear that F_i is locally connected if and only if all the prime ends of G_i are of the first kind [**4**, p. 65], and hence [**4**, p. 67] (a) implies (b). An elementary construction allows one to prove that (b) implies (c). (Borsuk originally showed directly that (a) implies (c).) It is clear that (b) or (c) implies (a).

By an abuse of language, we consider each point $v \in F_i$ as a prime end of the first kind, i.e., as a point with a representing embedding

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