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CONTRACTING EXTENSIONS AND CONTRACTIBLE GROUPS

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Wiener's classical tauberian theorem has been extended recently to some noncommutative, noncompact groups (see [1], [3], [8] and [10]). Our Theorems 1 and 2 are Wiener type theorems, and interest in them led to the study of contractible groups. It was rather surprising that all contractible Lie-groups are unipotent matrix groups (Theorem 3).

1. **Contracting group extensions.** A locally compact group N is *contractible* provided it has *enough contractions*, i.e., for any compact set $K \subset N$ and any neighborhood W of the identity in N, there is a homeomorphic automorphism $h \in \operatorname{Aut} N$ such that $hK \subset W$. The ordered pairs (K, W) form a directed set with respect to the relation $\leqq$, defined by $(K, W) \leqq (K', W')$ if and only if $K \subseteq K'$ and $W \supseteq W'$. For every $n = (K, W)$ choose a contraction h_n with $h_n K \subset W$, then $\{h_n\}$ is a net and for any compact set $K \subset N$ we have $\lim_n h_n K = \{e\}$ (e the neutral element of N).

A locally compact group G is a *contracting extension* of its normal subgroup N provided the set of restrictions to N of inner automorphisms of G contains enough contractions of N. Thus N must be contractible to admit contracting extensions. For example, if $G^{\cdot} \subseteq \operatorname{Aut} N$ is a locally compact group and contains enough contractions of N, then the semidirect product $G = G^{\cdot}$ Ⓢ N is a contracting extension of N.

If G is an extension of N and $G^{\cdot} = G/N$ is the corresponding factor group we will usually denote their elements respectively by x, ξ, $\dot{x}$, their (left) Haar measures by dx, $d\xi$, $d\dot{x}$, and their moduli by Δ, δ and $\Delta^{\cdot}$. We suppose that Weil's formula $dx = d\xi\, d\dot{x}$ holds.

Let us suppose for a moment that G is separable (i.e. has a countable basis of open sets). Then there exists a measurable cross-section σ of G with respect to N (cf. [9]); i.e., there is a measurable function $\sigma: G^{\cdot} \to G$ with $\sigma(\dot{x}) \in \dot{x} = xN$ and $\sigma(\dot{e}) = e$. Suppose further that there is a net $\{h_n\}$ of contractions of N as above, such that $\lim_n h_n(x)$ exists for locally almost

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[1] This article summarizes results from the author's dissertation which he prepared under the guidance of Horst Leptin, Bielefeld. The final version with proofs and worked out examples is being prepared for publication in J. Reine Angew. Math.