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DEFORMATIONS OF GROUP ACTIONS

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1. **Introduction.** The purpose of this note is to announce some results concerning the local structure of the space $A(G, M)$ of actions of a finite group G on a manifold M, endowed with the compact-open topology. We consider the questions of how nearly alike two actions must be if they are connected by a path in $A(G, M)$ and when two sufficiently close actions must be connected by a path.

In [5] R. Palais studied the space $D(G, M)$ of *differentiable* actions on a closed smooth manifold, endowed with the C^1 topology. His main result is the following (true, in fact, for any compact Lie group G):

THEOREM 1.1. *Let φ be in $D(G, M)$. Then there is a neighborhood U of φ in $D(G, M)$ and a continuous map $f: U \to$ Diffeo(M) (the latter with the C^1 topology also) such that $f(\varphi) = 1_M$ and $f(\psi) * \psi = \varphi$ for all ψ in U, where "$*$" denotes the usual action of* Diffeo(M) *on $D(G, M)$ by conjugation.*

Palais draws the following corollary (see also [6]):

COROLLARY 1.2. *If φ_t, $0 \leqq t \leqq 1$, is a path in $D(G, M)$, then there is a path f_t in* Diffeo(M) *such that $f_0 = 1_M$ and $\varphi_t = f_t * \varphi_0$, for all t.*

In particular, the space $D(G, M)$ is locally contractible, close actions are equivalent, and actions connected by a path are equivalent. Smoothness (as well as compactness) is necessary for these results. Thus in what follows we restrict attention to the topological and PL categories.

The results which follow constitute part of the author's thesis, written under the direction of Professor Frank Raymond at the University of Michigan. Details will appear elsewhere.

2. **When are G-isotopic actions equivalent?** Define a *G-isotopy* to be a level-preserving action in $A(G, M \times I)$. A PL G-isotopy is a G-isotopy in the space $A_{\mathrm{PL}}(G, M \times I)$ of PL actions. The following observation provides the typical examples of close, inequivalent, G-isotopic actions. The proof is analogous to the construction of the standard "Alexander isotopy."

PROPOSITION 2.1. *Let φ be in $A(G, D^n)$, where D^n is the unit n-disk. Then φ is G-isotopic to $C(\varphi \mid S^{n-1})$, the cone action over φ restricted to the boundary sphere S^{n-1}.*

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