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FUNCTIONAL ANALYSIS AND NONLINEAR DIFFERENTIAL EQUATIONS[1]

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1. The aim of this paper is to study the nonlinear differential equation

$$Ex = Nx \tag{1}$$

where N is a nonlinear operator in a real Hilbert space S, and E is a linear differential operator in S with preassigned linear homogeneous boundary conditions. The idea is to reduce the problem to a finite dimensional setting and this technique has been used by several authors. We use here a method due to Cesari [**4**]. This method has been extensively developed in the existence analysis of differential equations by Cesari, Hale, Locker, Mawhin and others. For a detailed bibliography one is referred to Cesari [**5**].

In this paper, by applying results from the theory of monotone operators, we show that, under suitable monotonicity hypotheses on N, the equation $Ex = Nx$ can be solved. In the present short presentation we restrict ourselves to the simplest hypotheses on E, N and S, even though the results obtained here hold under more general conditions.

2. Let S be the direct sum of the subspaces S_0 and S_1 and let $P:S \to S_0$ be a projection operator with null space S_1, and $H:S_1 \to S_1$ a linear operator such that (h_1) $H(I - P)Ex = (I - P)x$, x belonging to the domain of E. If y is a solution of (1), then $Ey = Ny$ implies $H(I - P)Ey = H(I - P)Ny$. Hence, $(I - P)y = H(I - P)Ny$; and finally

$$y = Py + H(I - P)Ny. \tag{2}$$

Thus, any solution of (1) is a solution of (2). If we also have that (h_2) $EPx = PEx$ and (h_3) $EH(I - P)Nx = (I - P)Nx$, then from (2) we derive

$$Ey = EPy + EH(I - P)Ny = PEy + (I - P)Ny.$$

Hence, $Ey - Ny = P(Ey - Ny)$. Thus, any solution y of (2) is a solution of (1) if and only if y satisfies

$$P(Ey - Ny) = 0. \tag{3}$$

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