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THE DEFICIENCY INDEX PROBLEM FOR ORDINARY SELFADJOINT DIFFERENTIAL OPERATORS

BY ALLEN DEVINATZ[1]

1. **Introduction.** In this lecture I shall attempt to survey the development during the past few years of the deficiency index theory for real selfadjoint ordinary differential operators. I shall not attempt to prove any theorems in detail since the proofs tend to be rather technically complicated. Indeed, I shall state only a very few theorems, either formally or informally, since the precise statements of many of the theorems in this field also tend to be rather technically complicated and in some cases rather lengthy. This state of affairs is, of course, not unusual when one is attempting to describe methods for the actual computation of real numbers, in this case nonnegative integers, in contrast, say, to existence or uniqueness statements. However, the ideas underlying many of the theorems and their proofs are often quite simple and I shall outline some of the ways in which the problem has been approached.

The deficiency index problem for ordinary selfadjoint differential operators, at least in the form that we now identify it, goes back to Hermann Weyl [**55**] around 1910, although in one guise or another it is present in investigations of selfadjoint boundary value problems going back a good deal longer. The work of Weyl as well as subsequent work indicates that there may be close connections between the deficiency index problem and the problem of describing the spectrum, at least qualitatively, of the selfadjoint extensions of the minimal operator associated with a formal ordinary selfadjoint differential operator. In some instances this connection is quite explicit in that knowledge of the deficiency index will give qualitative information about the spectra of selfadjoint extensions, and conversely. In other instances the connection takes a less explicit form in that the methods used to investigate one problem can also be used to investigate the other problem. Whether or not there are deeper underlying connections between these problems is not clear. I shall not attempt to deal with spectral problems here since this would take us too far afield, but shall direct my attention exclusively to the narrower problem indicated in the title.

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