## SOME TOPOLOGICAL APPROACHES TO THE FOUR COLOR PROBLEM

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The aim of this paper is to give a few observations with no proofs about the four color problem that are topological in nature. The approach is to define a 'local' 4 coloring on any 2-manifold and to measure the obstructions to a local coloring's being a desired type of coloring. All manifolds will be assumed to be oriented.

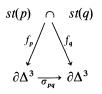
1. Local colorings. We begin with the definitions of four types of coloring that are possible on any surface.

(1) A four coloring of a triangulation K is a map  $f: K \to \partial \Delta^3$  which is simplicial, and maps triangles onto triangles. Here  $\partial \Delta^3$  is the tetrahedron, which has exactly 4 vertices.

(2) An *edge coloring* of K is a division of the edges of K into 3 sets, so that every triangle of K has an edge in each set. If we think of the sets as colors, every triangle has its edges 3 different colors.

(3) A heawood coloring is an assignment of +1 or -1 to every triangle of K so that the sum of the values on the triangles containing a point p is zero mod 3 for all vertices p of K.

(4) A local coloring is a collection of maps  $f_p:st(p) \to \partial \Delta^3$  and automorphisms  $\sigma_{pq}$  of  $\partial \Delta^3$  so that the following commutes.



In this definition, st(p) is the complex consisting of all triangles (and faces of them) containing p.  $\sigma_{pq}$  can be given as a permutation of the 4 vertices of  $\partial \Delta^3$ . We can restate the definition by saying we have a 4 coloring for each st(p) and, up to permutation of colors, the colorings of st(p) and st(q) agree on their overlap.

Next, we show how each type of 'coloring' induces the following one. Let  $f: K \to \partial \Delta^3$  be a four coloring.  $\partial \Delta^3$  has an edge coloring, so we get an edge coloring for K by pulling back the one on  $\partial \Delta^3$ . That is, the color of

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