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A SIMPLIFIED APPROACH TO THE COMPACTIFICATION OF MAPPINGS

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Let $f:X \to Y$ be an onto mapping (i.e., continuous function between Hausdorff spaces). If inverse images of compact sets are compact, f will be said to be *compact* and if the stronger condition that f is closed and inverse images of points are compact holds, f will be said to be *perfect*. A *compactification* (resp. *perfection*) of f is a compact (resp. perfect) mapping $f^*:X^* \to Y$ such that X is densely embedded in X^* and $f^* \mid X = f$.

Whyburn [9] introduced the notion of compactification of mappings and, by means of a "unified space" construction, showed that every mapping can be compactified. Specifically, for $f:X \to Y$, the unified space Z is the disjoint union of the underlying sets of X and Y such that $Q \subset Z$ is open provided that

(i) $Q \cap X$ and $Q \cap Y$ are open in X and Y respectively, and

(ii) for any compact set $K \subset Q \cap Y$, $f^{-1}[K] \cap (X - Q)$ is compact.

Then $r:Z \to Y$ defined by

$$r(x) = f(x) \quad \text{for } x \in X, \qquad r(x) = x \quad \text{for } x \in Y$$

is continuous and $r \mid \overline{X}:\overline{X} \to Y$ is a compactification of f. (Here $\overline{X}$ denotes the closure of X in Z.) In [12] and [13] Whyburn further investigated this construction and demonstrated its usefulness.

In 1969 Cain [3] presented a general construction that assigns to any mapping $f:X \to Y$, with Y regular, and any compactification $\tilde{X}$ of X, a compactification (in fact, a perfection) $f^*:X^* \to Y$ of f. Cain has produced two different constructions of his mapping compactification. The first [3] uses extensively the idea of filter space as developed by F. J. Wagner [8], while the second [4] uses rings of continuous functions and the assumption that Y is completely regular. Both constructions are considerably more complicated than the one to be presented below. Cain [4] has characterized his perfection (in the completely regular case) as the unique one with the property that there is a mapping h of X^* into $\tilde{X}$ which leaves both the points of X fixed and is such that for each $y \in Y$, $h \mid f^{*-1}(y)$ is a homeomorphism onto the subspace of $\tilde{X}$ consisting of all accumulation points of the inverse image of the neighborhood filter of y. He studies this

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