BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 79, Number 5, September 1973

AN APPLICATION OF MÖBIUS INVERSION TO A PROBLEM IN TOPOLOGICAL DYNAMICS

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Communicated by Walter Gottschalk, March 5, 1973

1. **Introduction.** Let X be a topological space and let ϕ be a homeomorphism of X onto X. The pair (X, ϕ) is called a cascade. A nonempty subset M of X is a minimal subset of (X, ϕ) if M is closed, $\phi(M) = M$, and no proper subset of M has these properties. Equivalently M $(M \neq \emptyset)$ is minimal if and only if for every x in M we have $\mathrm{Cl}\{\phi^n(x):n \in Z\} = M$. A homomorphism of (X, ϕ) into (Y, ψ) is a continuous map θ of X into Y such that $\theta \circ \phi = \psi \circ \theta$.

Let $K = \{z \in \boldsymbol{C}:|z| = 1\}$ and let (K, ϕ) be a cascade such that $\phi^n(x) = x$ implies $n = 0$. Then (K, ϕ) has exactly one minimal set which is either all of K or a Cantor subset C. We are only interested in the latter case, and we write

$$C = K \backslash \bigcup_{n=1}^{\infty} (a_n, b_n)$$

when (a_n, b_n) are counterclockwise open intervals in K and

$$[a_n, b_n] \cap [a_m, b_m] = \emptyset$$

whenever $n \neq m$. Note that $\phi[(a_i, b_i)] = (\phi(a_i), \phi(b_i)) = (a_j, b_j)$ for some $j \neq i$. Thus ϕ defines an equivalence relation on the complementary intervals $\{(a_n, b_n):n = 1, \ldots\}$. The restriction of ϕ to C is a homeomorphism of C onto C and produces a minimal cascade which we denote by (C, ϕ).

A cascade (X, ψ) on a compact Hausdorff space will be called an n-extension of (C, ϕ) if there exists an open n-to-one homomorphism of (X, ψ) onto (C, ϕ). If the number of equivalence classes of complementary intervals of C is finite, then for each positive integer n the number of isomorphism classes of minimal n-extensions is finite [**3**, Corollary 6.6]. Let $\mathscr{I}(C, \phi, n)$ denote this number. We consider the problem of determining $\mathscr{I}(C, \phi, n)$ or an asymptotic expression for it as n goes to infinity.

In the next section we present some combinatorial results which we applied to this problem, and in the last section we present our results on $\mathscr{I}(C, \phi, n)$. Proofs and tables of values can be found in [**1**].

2. **Combinatorial results.** Let N be a set with n elements, and let $\mathscr{S}_n$ be the symmetric group of all permutations acting on N. Let $\mathscr{S}_n^k$ be the set

AMS (MOS) subject classifications (1970). Primary 54H20, 05A15.