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LINEARIZATION STABILITY OF THE EINSTEIN EQUATIONS

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1. **Introduction.** An important problem in general relativity is the question of whether or not a solution of the linearized Einstein field equations (relative to a given background solution) actually approximates to first order a curve of exact solutions to the nonlinear equations. Here we announce that under certain geometrical conditions on the background solution the problem can be answered affirmatively; however, in certain exceptional cases the answer may be negative. In the affirmitive case we shall say that the background metric is *linearization-stable*.

Let ${}^{(4)}g$ be a Lorentz metric (signature $-, +, +, +$) on a 4-manifold V. The empty space Einstein field equations of general relativity are that the Ricci tensor of ${}^{(4)}g$ vanish:

$$\mathrm{Ric}({}^{(4)}g) = 0. \tag{1}$$

By an *infinitesimal deformation* we mean a 2-covariant symmetric tensor field ${}^{(4)}h$ which satisfies the linearized equations:

$$D\,\mathrm{Ric}({}^{(4)}g) \cdot {}^{(4)}h = 0, \tag{2}$$

where $D\,\mathrm{Ric}(\cdot)$ is the derivative of the map $\mathrm{Ric}(\cdot)$.

Assume that V has a compact orientable space-like hypersurface M and let g denote the induced Riemannian metric and k the second fundamental form. Our conditions are as follows:

(R)
(i) $\mathrm{tr}\, k$ (= trace of k) is constant on M.
(ii) If $k = 0$, g is not flat.
(iii) There are no nonzero vector fields X such that $L_X g = 0$ and $L_X k = 0$; L_X denotes the Lie derivative.

Under these conditions, every solution ${}^{(4)}h$ of (2) is tangent to a curve ${}^{(4)}g(\lambda)$ of exact solutions of (1); i.e. ${}^{(4)}g(0) = {}^{(4)}g$ in a tubular neighborhood of M in V, and $d{}^{(4)}g(\lambda)/d\lambda|_{\lambda=0} = {}^{(4)}h$ in this neighborhood.

The case of noncompact M is rather different. Here asymptotic conditions are necessary. For example, $k = 0$ and g the usual flat metric on $\boldsymbol{R}^3$

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