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A SMASH PRODUCT FOR SPECTRA

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ABSTRACT. We shall show that the smash product for pointed CW complexes induces a smash product $\wedge$ on the homotopy category of Adams's stable category with the following properties. $\wedge$ is coherently homotopy unitary (S^0), associative, and commutative, $\wedge$ commutes with suspension up to homotopy, and $\wedge$ satisfies a Kunneth formula.

Introduction. Precisely, we shall show that the homotopy category of a technical variant of Adams's stable category [1], a fraction category of CW prespectra equivalent to that of Boardman [3], [8], admits a symmetric monoidal structure in the sense of [4].

Whitehead's pairings of prespectra [9] and Kan and Whitehead's nonassociative smash product for simplicial spectra [6] were the first attempts at a smash product. Boardman gave the first homotopy associative, commutative, and unitary smash product in his stable category [3]. Adams has recently obtained a similar construction [2].

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1. **The interchange problem.** A S^k-prespectrum X consists of a sequence of pointed spaces $\{X_n \mid n \geqq 0\}$, together with inclusions $X_n \wedge S^k \to X_{n+1}$. Consider $S = \{S_n = S^{nk}\}$ as a ring with respect to the smash product of spaces. Then X is a right S-module.

Construction of a homotopy associative smash product for S^k-prespectra requires permutations π of $S^k \wedge \cdots \wedge S^k$. Since S is not strictly commutative, but only graded homotopy commutative, this requires defining suitably canonical maps of degree -1 (for k odd) and homotopies $\pi \simeq \pm\mathrm{id}$.

We avoid sign problems by using S^4-prespectra and define *canonical homotopies* H_π as follows. Make the standard identifications

$$S^{4k} \cong S^4 \wedge \cdots \wedge S^4 \cong I^4/\partial I^4 \wedge \cdots \wedge I^4/\partial I^4 \cong I^{4k}/\partial I^{4k} \cong D^{4k}/S^{4k-1}.$$

Then π simply permutes factors of $C^2 \times \cdots \times C^2$. Hence $\pi \in SU(2k)$.

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