BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 79, Number 5, September 1973

# BRANCHED COVERINGS OF GRAPH IMBEDDINGS

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Communicated by Hyman Bass, March 12, 1973

This announcement outlines a reformulation of W. Gustin's combinatorial theory of current graphs [3] and J. W. T. Youngs' extension of that theory to vortex graphs [8] into the topological context of covering spaces and branched covering spaces. Whereas certain restrictions imposed by Gustin and Youngs were convenient in obtaining minimal imbeddings of complete graphs, leading to the solution of the Heawood map-coloring problem (see Ringel and Youngs [6]), the present relaxation of those restrictions leads to a more general method of constructing minimal and other imbeddings of graphs.

1. **Current graphs.** A *rotation system* on a graph $K$ is a map $\varphi$ that assigns to each vertex $v$ a cyclic permutation $\varphi_v$, called the *rotation at* $v$, of the set of all vertices adjacent to $v$. The pair $(K, \varphi)$ is called a *rotation* graph.

A rotation system acts as a permutation on the set of oriented edges of its graph. The orbits under the cyclic group generated by this permutation are called *rotation circuits*. If each rotation circuit is regarded as the boundary of a polygon, then the graph is imbedded in an oriented surface so that the rotation at each vertex corresponds to the surface orientation.

The present authors call this imbedding the *schematic imbedding*, reflecting Ringel's terminology ("scheme") for his way (see [5]) of describing a rotation system in his pioneering attack on the Heawood map-coloring problem. The underlying concept appears to be an invention of L. Heffter [4]. J. Edmonds has observed that it leads to an algorithm for computing the genus of any graph. The schematic imbedding and the Edmonds algorithm are described in detail by Youngs [7]. The algorithm is prohibitively time-consuming, even on a very fast computer, and it has firmly resisted all attempts at acceleration. It would be interesting to know whether any essential speed-up is possible.

A *current assignment* in a group $G$ for a rotation graph $(K, \varphi)$ is a map $\alpha$ from the set of oriented edges of $K$ to the group $G$ such that the following rules hold.

*AMS (MOS) subject classifications* (1970). Primary 05C10, 55A15; Secondary 05C25, 55A10, 05C15.

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[2] The work of the second author was partially supported by NIH Division of Research Resources, Biotechnology Resources Branch Grant #RR00291.